Implicit Differentiation (Not in textbook)

We are now masters at finding $y'$ when $y = f(x)$ (explicit function)

But what if we can't solve for $y$ explicitly?!

Ex: (1) $x^2 + y^2 = 1$

(2) $2y + y^{23} - x^3 = 20$

(3) $3x^3y^3 + x^2y + 13x = 12e^x$

Taking derivatives of expressions like these is called implicit differentiation and is really just chain rule!

Remember: $y$ is a function of $x$.

Ex: What's $y'$ when $x^2 + y^2 = 1$?

Solution: Differentiate everything (Don't forget to use chain rule on $y$ terms)

$$x^2 + y^2 = 1 \Rightarrow 2x + 2y \cdot y' = 0$$

$$\Rightarrow 2y \cdot y' = -2x$$

$$\Rightarrow y' = \frac{-2x}{2y} = \frac{-x}{y}$$

↑

Finish by solving for $y'$. 
Ex: What's y' when \(2y + y^{23} - x^3 = 20\) ?

Solution: Take derivatives everywhere!

\[
2y + y^{23} - x^3 = 20 \implies 2y' + 23y^{22}y' - 3x^2 = 0
\]

\[\implies y' \left(2 + 23y^{22}\right) = 3x^2\]

\[\implies y' = \frac{3x^2}{2 + 23y^{22}}\]

Ex: What's y' when \(3x^3y^3 + x^2y + 13x = 12e^x\) ?

Solution: Derivatives! We'll need product rule...

\[
3(3x^2y^3 + x^2 \cdot 3y^2y') + (2x \cdot y + x^2 \cdot y') + 13 = 12e^x
\]

\[\implies 9x^2y^3 + 9x^3y^2y' + 2xy + x^2y' + 13 = 12e^x\]

\[\implies y' \left(9x^3y^2 + x^2\right) = 12e^x - 13 - 2xy - 9x^2y^3\]

\[\implies y' = \frac{12e^x - 13 - 2xy - 9x^2y^3}{9x^3y^2 + x^2}\]