Chapter 6 - Differentiation Techniques

Goal: Develop faster methods for computing \( f'(x) \)
(since, ya know, using the definition sucks!)

§ 6.1 - Preliminary Methods

**The Constant Rule:** If \( f(x) = k \) (\( k \) is a constant), then \( f'(x) = 0 \)

To see why, use the definition:

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{k-k}{h} = 0
\]

**Ex:** \( f(x) = \sqrt{3} \Rightarrow f'(x) = 0 \), \( g(x) = \pi + 7 \Rightarrow g'(x) = 0 \).

What about powers of \( x \)?

In § 5.4 we saw that

\[
\begin{align*}
f(x) &= x \Rightarrow f'(x) = 1 \\
f(x) &= x^2 \Rightarrow f'(x) = 2x \\
f(x) &= x^3 \Rightarrow f'(x) = 3x^2
\end{align*}
\]

**The Power Rule:** If \( f(x) = x^n \), then \( f'(x) = nx^{n-1} \).

**Ex:**

1. \( f(x) = x^4 \Rightarrow f'(x) = 4x^3 \) \( \text{(Proof in text)} \)

   \( \sim \) This was our guess back in § 5.4!

2. \( f(x) = x^n \Rightarrow f'(x) = nx^{n-1} \)

3. \( f(x) = \sqrt{x} \) \( (= x^{\frac{1}{2}}) \Rightarrow f'(x) = \frac{1}{2} x^{-\frac{1}{2}} \)

4. \( f(x) = \frac{1}{x^3} \) \( (= x^{-3}) \Rightarrow f'(x) = -3x^{-4} \)
(5) \( f(x) = x^\pi \Rightarrow f'(x) = \pi x^{\pi-1} \)

(6) \( f(x) = 8x^2 \Rightarrow f'(x) = ?? \)  
What do we do when a constant is out front?

**The Coefficient Rule:** The derivative of \( K \cdot f(x) \) is \( K \cdot f'(x) \)  
(i.e., just ignore the coefficient!)

**Ex:**  
(1) \( f(x) = 8x^2 \Rightarrow f'(x) = 8(2x) = 16x \)

(2) \( f(x) = \frac{6}{x} \)  
\( (= 6x^{-1}) \Rightarrow f'(x) = -6x^{-2} = \frac{-6}{x^2} \)

(3) \( f(x) = 7\sqrt{x^7} \)  
\( (= 7x^{4/3}) \Rightarrow f'(x) = \frac{7(4/3)x^{4/3-1}}{3} = \frac{28}{3}x^{4/3} \)

(4) \( f(x) = \pi x^e \Rightarrow f'(x) = \pi e x^{e-1} \)

(5) \( f(x) = 2x - 3x^5 \Rightarrow f'(x) = ?? \)  
How do we handle sums/differences?

**The Sum/Difference Rule:** The derivative of \( f(x) \pm g(x) \) is \( f'(x) \pm g'(x) \).  
(i.e., we do the derivative term-by-term)

**Ex:**  
(1) \( f(x) = 2x - 3x^5 \Rightarrow f'(x) = 2 - 15x^4 \)

(2) \( f(x) = 12x^4 + 6\sqrt{x} - 1 \Rightarrow f'(x) = 48x^3 + 3x^{-1/2} \)
(3) \( f(x) = \frac{x^3 + x^2}{x} \)

We first rewrite \( f(x) = \frac{x^3}{x} + \frac{x^2}{x} = x^2 + x \).

Now \( f'(x) = 2x + 1 \)

(4) \( f(x) = (x-2)^3 \)

We don't know how to handle products (yet!) so let's first expand:

\[ f(x) = (x-2)^3 = x^3 - 6x^2 + 12x - 8. \]

Thus, \( f'(x) = 3x^2 - 12x + 12 \)

Ex: Find the values of \( x \) at which the tangent line to \( f(x) = x^3 - x^2 \) is horizontal.

Solution: Horizontal tangent means \( f'(x) = 0 \).

\[ f'(x) = 3x^2 - 2x = x(3x - 2) = 0. \]

\[ \Rightarrow x = 0 \quad \text{or} \quad 3x - 2 = 0, \text{ so } x = \frac{2}{3} \]