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AN OVERVIEW OF THE COLLECTED WORKS OF PROFESSOR DRAGOSLAV ŠILJAK

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Preface

Dragoslav D. Šiljak was born in Belgrade, Serbia, on September 10, 1933. He started elementary school in the Fall of 1940 and enjoyed a few months of school, when, suddenly, one morning in the Spring of 1941 a barrage of bombs engulfed the apartment house where his family resided; the Second World War arrived in Serbia, and especially Belgrade, with fury and destruction. For the next four years, his life and the life of his family would hang in the balance. Hunger, extreme cold in Winter without heat, and persistent bombing of Belgrade, made fear for life always present. To survive, the family would escape from the city into the countryside to stay and work with their relatives, only to find themselves in the middle of the guerilla war that raged in the Serbian heartland. The schools were closed in the city, and the only education he could get was in a rural area school, which was open seldom during short stretches of peace.

After the end of the war, life started to improve steadily, schools were open, and Šiljak went back to school. At fourteen, he started competitive swimming and water polo at the local club, balancing study and practice almost daily. In his senior year, at eighteen, he made the Yugoslav national water polo team that won the silver medal at the 1952 Olympic Games in Helsinki, Finland. A month later, he entered the University of Belgrade to study electrical engineering and continue the balancing act of study and practice in an increasingly demanding professional sport. He made the national team again, which won the world cup in 1953, in Nijmegen, The Netherlands. At the University, Šiljak liked mathematics, earning the highest grades in math courses. In his senior year he took an exciting course on the theory of automatic control that was taught by Professor Dušan Mitrović. From then on, his professional interest was set for life. He once again made the Yugoslav Olympic team for the 1960 Olympic Games in Rome, Italy, but the team took a disappointing fourth place.

Šiljak started doing research in 1957, while working on his BS thesis under Professor Mitrović in the Department of Electrical Engineering at the University of Belgrade. The topic of the thesis was a controller design in the parameter space for sampled-data systems. He applied Mitrović's method to obtain stability regions in the space of controller parameters, and published his first IEEE Transactions paper in 1961, based on his MS thesis on control of sampled-data systems with time-delay. He continued this line of research by fusing Mitrović's method with Neimark's D-decomposition, and published his results in a sequence of three IEEE Transactions papers on the control of continuous, sampled-data, and nonlinear systems in the parameter space. An interesting novelty in this work was the introduction of Chebyshev's polynomials into the computation of stability regions in the parameter space. Equally interesting was Šiljak's proof of the shading rule in D-decompositions, which was crucial in identifying the root distribution of characteristic equations in the parameter space. Šiljak's contributions to the parameter space design of control systems, which served as a major part of his PhD dissertation in 1963, were not witnessed by his adviser Professor Mitrović, due to his tragic death in 1961. In 1963, Šiljak was promoted to the rank of Docent. Soon after promotion, he accepted an invitation by G. J. Thaler to visit Santa Clara University, and left for USA in 1964.

Immediately after arriving at Santa Clara, Šiljak started building a research base centered around parameter space methods for the design of control systems. He interacted with Thaler in their common research interest, and in 1965 they obtained a research grant from NASA Ames Research Center to develop the parameter space approach. This research activity attracted the attention of S. M. Seltzer from NASA Marshal Space Center, where Šiljak's parameter space methods were used in the control design of spacecraft, in particular, the large booster Saturn V that carried man to the moon. NASA continued to support Šiljak's joint works with Thaler and Seltzer well into the l970's, resulting in a number of improvements that made the parameter space design competitive with well-established classical designs based on root-locus, Bode, Nyquist and Nichols diagrams.

Below we present concise description of important problems (in our view) that were formulated and solved by Professor Šiljak. The problems are grouped in ten main research areas.

1 Parameter Space Design of Control Systems

To highlight a few interesting aspects of the parameter space approach, let us consider a real polynomial

$$f(s) = \sum_{k=0}^{n} a_k(p)s^k, \qquad a_n \neq 0$$

which is a characteristic polynomial of a linear control system, whose coefficients $a_k(p)$ are dependent on a parameter vector p. When we express the complex variable s as

$$s = -\omega_n\zeta + j\omega_n\sqrt{1-\zeta^2}$$

the real and imaginary parts of the polynomial can be expressed in the compact form

$$R(\zeta,\omega_n;p) \equiv \sum_{k=0}^{n} (-1)^k a_k(p)\omega_n^k T_k(\zeta)$$

$$I(\zeta,\omega_n;p) \equiv \sum_{k=0}^{n} (-1)^{k+1} a_k(p)\omega_n^k \sqrt{1-\zeta^2}U_k(\zeta)$$

where $T_k(\zeta)$ and $U_k(\zeta)$ are Chebyshev polynomials of the first and second kind.

If the complex variable is replaced by

$$s = \sigma + j\omega$$

R and I can be reformulated as

$$R(\sigma,\omega;p) \equiv \sum_{k=0}^{n} a_k(p)X_k(\sigma,\omega)$$

$$I(\sigma,\omega;p) \equiv \sum_{k=0}^{n} a_k(p)Y_k(\sigma,\omega)$$

where the polynomials X_k and Y_k, which were introduced by Šiljak, are computed by the recurrence formulas

$$X_{k+1} - 2\sigma X_k + (\sigma^2 + \omega^2)X_{k-1} = 0$$
$$Y_{k+1} - 2\sigma Y_k + (\sigma^2 + \omega^2)Y_{k-1} = 0$$

with $X_0 = 1$, $X_1 = \sigma$, $Y_0 = 0$ and $Y_1 = \omega$.

Both Chebyshev's and Šiljak's forms of R and I play a useful role in delineating stability regions in the parameter space, which correspond to desired root regions in the complex $s-$plane. When Šiljak's polynomials are used, boundaries of the regions are defined by the equations

$$\begin{aligned} &\text{complex root boundaries:} && R(\sigma, \omega; p) = 0 \\ & && I(\sigma, \omega; p) = 0 \\ &\text{real root boundaries:} && a_0(p) = 0 \\ & && a_n(p) = 0 \end{aligned}$$

By plotting the boundaries in the parameter space, the designer can establish a direct correlation between root regions in the complex plane and adjustable parameters appearing in the coefficients of the corresponding characteristic equation. Obviously, stability regions can be easily visualized when two or, at most, three parameters are allowed to be free. In the early stages of the parameter space development, this was a considerable advantage over classical single parameter methods, especially because parameter space boundaries could be plotted accurately using modest computing machines, or even a hand calculator.

When parameter vector p has dimension larger than three, stability regions are difficult to visualize, and Šiljak concluded that the problem of robust stability analysis is one of *interpretation*. In a joint work with J. S. Karmarkar, he imbedded a sphere within design constraints in the parameter space and formulated a Chebyshev-like minimax problem,

$$\begin{aligned} &\textit{minimize} && -\rho \\ &\textit{subject to} && h_i(p) - \rho \geq 0, \qquad i = 1, 2, \ldots, r \end{aligned}$$

where $h_i(p)$ are continuous functions representing the design constraints, and ρ is the radius of the sphere. This allowed for the use of mathematical programming algorithms to maximize the radius, and get the largest estimate of the design region, which can be readily interpreted in the parameter space. Since there is a considerable freedom in choosing design constraints, using the computed sphere the designer can choose feedback gains which guarantee robust stability among other relevant performance specifications. The paper with Karmarkar was presented at the 1971 Allerton conference, where in the title they coined the term "computer-aided control design." This work has been recognized in the literature as a precursor to the modern use of convex optimization and mathematical programming in control system design.

Among the interesting aspects of Šiljak's polynomials (e.g., that they obey the Pascal triangle) is the fact that they played a significant role in a gradient root solving scheme which Šiljak initiated with P. Kokotović. An important feature of the root solving algorithm is that, in minimizing a Lyapunov-like function,

$$V(\sigma,\omega) = R^2(\sigma,\omega) + I^2(\sigma,\omega),$$

the algorithm converges *globally* and *quadratically* to a root in the complex plane independently of the disposition of the polynomial equation, the distribution of its roots, or their distance from the starting point. J. Stolan improved Šiljak's root-solver and compared it with the modern versions of the algorithms of Laguerre, Jenkins-Traub, and MATLAB to demonstrate the superiority of the improved Šiljak's scheme in solving difficult benchmark problems.

2 Positivity of Uncertain Polynomials

Let us consider a real polynomial $g(s)$ and define positivity as

$$g(s) \equiv \sum_{k=0}^{n} g_k s^k > 0, \qquad \forall s \in \mathbb{R}_+.$$

To provide an algebraic criterion for the positivity of $g(s)$ in terms of the coefficients g_k, Šiljak formulated the *Modified Routh Array*,

$$\begin{array}{lllllll}
r_0 & = & (-1)^n g_n & (-1)^{n-1} g_{n-1} & \dots & -g_1 & g_0 \\
r_1 & = & (-1)^n n g_n & (-1)^{n-1}(n-1) g_{n-1} & \dots & -g_1 & \\
\vdots & & & & & & \\
r_{2n} & = & g_0 & & & &
\end{array}$$

and proved that the number κ of positive zeros of $g(s)$ is

$$\kappa = n - V(r_0, r_1, \dots, r_{2n}),$$

where V is the number of sign variations in the coefficients of the first column of the array. In particular, the *positivity* of $g(s)$ follows from setting $g_0 > 0$ and requiring that $\kappa = 0$.

Šiljak reformulated Popov's frequency inequality in terms of polynomial positivity and provided a test for absolute stability of the Lur'e-Postnikov nonlinear systems, which had the same numerical simplicity as the Routh-Hurwitz test for stability of linear time-invariant systems. He extended the polynomial positivity test to determine *positive realness* of real rational functions and matrices. At the same time, he generalized the test for positivity of real polynomials on the unit circle, which led to efficient numerical algorithms for testing *circle positive realness* of rational functions and matrices.

In 1979, Šiljak extended his positivity algorithm to testing *stability of two-variable polynomials*, which is required in the design of two-dimensional digital filters. A real two-variable polynomial

$$h(s,z) = \sum_{j=0}^{n} \sum_{k=0}^{m} h_{jk} s^j z^k$$

where $s, z \in \mathbb{C}$ are two complex variables, is said to be stable if

$$h(s,z) \neq 0, \qquad \left\{ s \in \mathbb{C}_-^C \right\} \cap \left\{ z \in \mathbb{C}_-^C \right\}.$$

$\mathbb{C}_-^C$ is the complement of $\mathbb{C}_- = \{ s \in \mathbb{C} : \operatorname{Re} s < 0 \}$ - the open left half of complex plane $\mathbb{C}$. As shown by H. G. Ansel in 1964, $h(s,z)$ is stable if and only if

$$h(s,1) \neq 0, \qquad \forall s \in \mathbb{C}_-^C$$
$$h(i\omega, z) \neq 0, \quad \forall z \in \mathbb{C}_-^C$$

Šiljak showed that these conditions are equivalent to easily testable conditions

$$f(s) \neq 0, \quad \forall s \in \mathbb{C}_-^C$$
$$g(\omega) > 0, \quad \forall \omega \in \mathbb{R}_+$$
$$H(0) > 0$$

where $f(s) = h(s,1)$, $g(\omega) = \det H(\omega)$, and $H(\omega)$ is the corresponding Hermite matrix. In this way, to test a two-variable polynomial for stability, all one needs is to compute two Routh arrays, one standard array for the stability of $f(s)$ and one modified array for the positivity of $g(\omega)$, and determine if a numerical matrix $H(0)$ is positive definite. The extensions of the test to stability with respect to two unit circles, as well as the imaginary axis and unit circle, were shown to be equally elegant and efficient. Over the years, the efficiency of Šiljak's stability test has been used as a measuring stick by people who have proposed alternative numerical tests for stability of two-variable polynomials.

For either modeling or operational reasons, control system parameters are not known precisely. Stability analysis must account for parametric uncertainty in order to provide useful information about system performance. This fact motivated Šiljak to initiate a study of robust absolute stability of the Lur'e-Postnikov's nonlinear control system

$$\mathbf{S}: \quad \begin{aligned} \dot{x} &= Ax + bu \\ y &= c^T x \\ u &= -\phi(t, y) \end{aligned}$$

where the vectors b and c are parts of an uncertain parameter vector p. He rewrote Popov's inequality as polynomial positivity

$$g(\omega, p) \equiv \det(\omega^2 I + A^2)\{2k - c^T A(\omega^2 I + A^2)b - \\ - \left[c^T A(\omega^2 I + A^2)b\right]\} > 0, \qquad \forall \omega \in \mathbb{R}_+$$

and showed that stability region $\mathbf{P}$ in the parameter space is *convex* with respect to either b or c, and that the boundary of the region is an envelope defined by the two equations

$$g(\omega, p) = 0, \qquad \frac{\partial}{\partial \omega} g(\omega, p) = 0$$

accompanied by two additional equations

$$g_0(p) = 0, \qquad g_n(p) = 0.$$

Again, Šiljak and Karmarkar interpreted absolute stability regions in the parameter space using a Chebyshev-like method of inequalities.

When in the 1980's the work of Kharitonov revived interest in robust stability, Šiljak returned to stability under parametric perturbations and wrote his 1989 survey paper reviewing the field of robust stability in the parameter space. With his son Matija Šiljak (then MSEE student at Stanford University) he reconsidered positivity of uncertain polynomials and polynomial matrices, and with his former student Dušan Stipanović used Bernstein polynomials to solve the problems of robust D-stability, positive realness of real rational functions and matrices, as well as stability of two-variable uncertain polynomials having polynomic uncertainty structures.

A comprehensive presentation of parameter space control design was published by Šiljak in his first monograph *Nonlinear Systems* (Wiley, 1969), where a number of interesting topics were introduced and developed. Šiljak's results on stability and positivity of polynomials have been referred to and used by many people, most notably by V. A. Yakubovich, E. I. Jury, Ya. Z. Tsypkin, G. J. Thaler, A. Tesi, I. B. Yunger, A. Vicino, B. T. Polyak, S. P. Bhattacharyya, Y. Bistritz, V. I. Skorodinskii, S. M. Seltzer, M. Dahleh, R. K. Yedavalli, X. Hu, J. Gregor, L. H. Keel, and D. Henrion.

3 Large-Scale Systems: Connective Stability

In the early 1970's, Šiljak started his research on large-scale systems that lasts to the present day. At the outset, he argued that structural uncertainty is the central problem of stability of large interconnected systems. At the 1971 Allerton conference, Šiljak introduced the concept of *connective stability*, which requires that the system remains stable in the sense of Lyapunov under structural perturbation whereby subsystems are disconnected and again connected in unpredictable ways during operation. In controlling large interconnected systems, connective stability has been particularly useful because, by accident or design, the controlled systems change their interconnection structure.

To sketch the concept of connective stability, let us consider a dynamic system

$$\mathbf{S}: \quad \dot{x} = f(t, x)$$

where $x(t) \in \mathbb{R}^n$ is the state of $\mathbf{S}$ at time $t \in \mathbb{R}$, and the function $f : \mathbb{R}^{n+1} \to \mathbb{R}^n$ is piecewise continuous in both arguments. We assume that $f(t, 0) = 0$ for all $t \in \mathbb{R}$ and that $x = 0$ is the unique equilibrium of $\mathbf{S}$.

To study the stability of the equilibrium under structural perturbations, we assume that the system is $\mathbf{S}$ decomposed as

$$\mathbf{S}: \quad \dot{x}_i = g_i(t, x_i) + h_i(t, x), \qquad i \in \mathbb{N}$$

which is an interconnection of N subsystems

$$\mathbf{S}_i\!: \quad \dot{x}_i = g_i(t, x_i), \qquad i \in \mathbb{N}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state of the i-th subsystem $\mathbf{S}_i$, function $h_i : \mathbb{R}^{n+1} \to \mathbb{R}^{n_i}$ is the interconnection of subsystem $\mathbf{S}_i$ with the rest of the system $\mathbf{S}$, and $\mathbb{N} = \{1, 2, \ldots, N\}$. To describe the structure of $\mathbf{S}$, we represent the interconnection functions as

$$h_i(t, x) \equiv h_i(t, \bar{e}_{i1} x_1, \bar{e}_{i2} x_2, \ldots, \bar{e}_{iN} x_N), \qquad i \in \mathbb{N}$$

where the binary numbers $\bar{e}_{ij}$ are elements of the $N \times N$ *fundamental interconnection matrix* $\bar{E} = (\bar{e}_{ij})$ defined by

$$\bar{e}_{ij} = 1, \quad x_j \text{ occurs in } h_i(t, x)$$
$$\bar{e}_{ij} = 0, \quad x_j \text{ does not occur in } h_i(t, x)$$

which is the standard occurrence matrix, meaning $\bar{e}_{ij} = 1$ if $\mathbf{S}_j$ acts on $\mathbf{S}_i$, or $\bar{e}_{ij} = 0$ if it does not. *Structural perturbations* of $\mathbf{S}$ are described by the $N \times N$ interconnection matrix $E = (e_{ij})$ where the elements $e_{ij} : \mathbb{R}^{n+1} \to [0, 1]$ are piecewise continuous in t and continuous in x. Matrix E is generated by $\bar{E}$ (denoted $E \in \bar{E}$) when $\bar{e}_{ij} = 0$ if and only if $e_{ij}(t, x) \equiv 0$, and describes quantitative and qualitative changes in the interconnection structure of $\mathbf{S}$ defined by $\bar{E}$.

A system $\mathbf{S}$ *is said to be connectively stable if its equilibrium is globally asymptotically stable for all* $E \in \bar{E}$.

Conditions for connective stability were obtained by Šiljak using the Matrosov-Bellman concept of vector Lyapunov functions and the theory of differential inequalities developed by V. Lakshmikantham and S. Leela. With each subsystems one associates a scalar function $\nu_i : \mathbb{R}^{n_i+1} \to \mathbb{R}_+$, which is a continuous function that satisfies a Lipschitz condition in x_i with a constant $\kappa_i > 0$. We also require that subsystem functions $\nu_i(t, x_i)$ satisfy the inequalities

$$\phi_{i1}(\|x_i\|) \leq \nu_i(t, x_i) \leq \phi_{i2}(\|x_i\|)$$
$$D^+\nu_i(t, x_i)_{\mathbf{S}_i} \leq -\phi_{i3}(\|x_i\|), \qquad \forall (t, x_i) \in \mathbb{R}^{n_i+1}$$

where $\phi_{i1}, \phi_{i2} \in \mathcal{K}_\infty, \phi_{i3} \in \mathcal{K}$ are Hahn's functions.

As for the interconnections, we assume that there are numbers ξ_{ij} such that $\xi_{ij} \geq 0$ $(i \neq j)$, and

$$\|h_i(t, x)\| \leq \sum_{j=1}^{N} \bar{e}_{ij}\xi_{ij}\phi_{i3}(\|x_j\|), \qquad \forall (t, x) \in \mathbb{R}^{n+1}.$$

Finally, we form a scalar Lyapunov function

$$V(t, x) = d^T\nu(t, x),$$

where $\nu : \mathbb{R}^{n+1} \to \mathbb{R}^N$ is the vector Lyapunov function $\nu = (\nu_1, \nu_2, \ldots, \nu_N)^T$ and $d \in \mathbb{R}^N_+$ is a constant vector with positive components, and compute a differential inequality

$$D^+\nu(t, x)_{\mathbf{S}} \leq -d\bar{W}\phi_3(x), \qquad \forall (t, x) \in \mathbb{R}^{n+1}$$

where $\phi_3 = (\phi_{13}, \phi_{23}, \ldots, \phi_{N3})^T$, and the so-called $N \times N$ *aggregate matrix* $\bar{W} = (\bar{w}_{ij})$ is defined by

$$\bar{w}_{ij} = 1 - \bar{e}_{ii}\kappa_i\xi_{ii}, \quad i = j$$
$$\bar{w}_{ij} = -\bar{e}_{ij}\kappa_i\xi_{ij}, \qquad i \neq j$$

The system $\mathbf{S}$ *is connectively stable if the matrix* $\bar{W}$ *is an M-matrix.*

A comprehensive early account of connective stability and large-scale systems has been provided by Šiljak in his book *Large-Scale Dynamic System: Stability and Structure* (North Holland, 1978). The initial results have been greatly improved by works of many people, notably by M. Ikeda, G. S. Ladde, Y. Ohta, Lj. T. Grujić, V. Lakshmikantham, S. Leela, V. A. Matrosov, A. A. Martynyuk, X. Liu, and D. M. Stipanović. Many of the improvements and generalizations have used the well-known stability results obtained by F. N. Bailey, A. Michel, and M. Araki.

4 Competitive-Cooperative Systems

Soon after proposing the connective stability concept, Šiljak observed the fact that *monotone functions* (known also as Kamke's functions) in differential inequalities appear as gross-substitute excess demand functions in models of competitive equilibrium in multiple-market systems, as well as models of mutualism in Lotka-Volterra population models, and Richardson's model of arms race. Using this fact, he published a paper in *Nature* in 1974 proposing M-matrix conditions for stability of multi-species communities, arguing at the same time that these conditions provide the proper context for resolving the stability vs. complexity problem in model ecosystems; the problem was studied extensively by many people including W. R. Ashby, R. C. Lewontin, R. M. May, G. S. Ladde, V. Grimm, B. S. Goh, and S. L. Pimm. In a series of papers in the 1970's and in the 1978 monograph on large-scale systems, Šiljak applied M-matrices to stability of the models of competition and co-operation in economics, multispecies communities, and arms race. In this context he introduced *nonlinear matrix systems*,

$$\mathbf{S}: \quad \dot{x} = A(t,x)x,$$

where the coefficients of the $n \times n$ system matrix $A = (a_{ij})$ were defined as

$$a_{ij}(t,x) = -\delta_{ij}\varphi_i(t,x) + e_{ij}(t,x)\varphi_{ij}(t,x)$$

δ_{ij} is the Kronecker symbol, and $\varphi_i(t,x)$ and $\varphi_{ij}(t,x)$ are continuous functions in both arguments. By properly bounding these functions, Šiljak used again the M-matrix to establish the connective stability of nonlinear matrix systems.

The obtained results have been extended by Ladde and Šiljak to stability analysis of population models in random environments described by stochastic equations of the Itô type,

$$dx = A(t,x)xdt + B(t,x)xdz,$$

where $z \in \mathbb{R}$ is a random Wiener process with $E\left\{[z(t_1) - z(t_2)]^2\right\} = |t_1 - t_2|$. The elements $a_{ij}(t,x)$ and $b_{ij}(t,x)$ of nonlinear time-varying matrices $A = (a_{ij})$ and $B = (b_{ij})$ have been expressed in terms of the piecewise continuous elements of interconnection matrices $E = (e_{ij})$ and $L = (l_{ij})$, respectively. The interconnection matrices were used to model uncertain structural changes in both the deterministic and stochastic interactions among species in the community, and serve as a basis for connective stability of ecosystems in a stochastic environment.

This line of research has been successfully generalized by G. S. Ladde for stability analysis of compartmental and chemical systems, systems with diffusion, and hybrid systems. Expositions of numerous results obtained by many people in the field of monotone systems (also known as positive systems) have been provided in monographs by G. S. Goh, and by E. Kaszkurewicz and A. Bhaya. In a seminal survey paper on ecosystem stability, V. Grimm and C. Wissel included connective stability in their survey and extensive discussion of concepts of ecological stability.

5 Parametric Stability

During his study of Lotka-Volterra models, Šiljak observed that structural perturbations result in changes of the nonzero equilibrium of the model. He called this phenomenon *moving equilibrium* and pointed out that the standard preconditioning of stability analysis, whereby the equilibrium is shifted to the origin, may obscure the fact that the original equilibrium is not fixed and its location is uncertain, causing liability in the end results. Šiljak, in collaboration with M. Ikeda and Y. Ohta, formulated a new concept of *parametric stability* which addresses simultaneously the twin problem of existence and stability of a moving equilibrium.

When motions of a time-invariant dynamic system

$$\mathbf{S}: \quad \dot{x} = f(x, p)$$

with state $x(t) \in \mathbb{R}^n$ at time $t \in \mathbb{R}$, depend on a constant parameter vector $p \in \mathbb{R}^l$, we assume that there is a stable equilibrium state $x^e(p^*) \in \mathbb{R}^n$ for a nominal parameter value $p^* \in \mathbb{R}^l$.

We say that system $\mathbf{S}$ *is parametrically stable at* p^* *if there is a neighborhood* $\Omega(p^*) \subset \mathbb{R}^l$ *such that:*

(i) *an equilibrium* $x^e(p^*) \in \mathbb{R}^n$ *exists for any* $p \in \Omega(p^*)$*, and*

(ii) *equilibrium* $x^e(p)$ *is stable for any* $p \in \Omega(p^*)$*.*

The concept has been applied to Lotka-Volterra models relying on M-matrix theory. A challenging parametric absolute stability problem has been solved for Lur'e-Postnikov systems, which required special care due to uncertain nonlinearities. Two distinct solutions were obtained in collaboration with T. Wada, one via Popov's inequality and the other using linear matrix inequalities. Ohta and Šiljak presented a scheme to quadratically stabilize uncertain systems with a moving equilibrium using linear feedback. Recently, the concept has been generalized to include stability of moving invariant sets and dynamical systems on time scales by V. Lakshmikantham and S. Leela. A. I. Zečević and Šiljak showed how stabilization of the moving equilibrium can be accomplished applying linear feedback control within the framework of convex optimization involving linear matrix inequalities.

6 Decentralized Control of Complex Systems

Complexity is a subjective notion, and so is the notion of complex systems. Šiljak argued that in controlling large complex systems the accumulated experience suggests that the following features are essential:

Dimensionality

Uncertainty

Information Structure Constraints

Most of the methods and results along these lines, which were obtained in the field of decentralized control before 1991, have been surveyed and presented by Šiljak in his monograph *Decentralized Control of Complex Systems* (Academic Press, 1991).

To describe some of the most interesting ideas in this context, let us consider a linear time-invariant dynamic system

$$\mathbf{S}: \quad \dot{x} = Ax + Bu$$
$$y = Cx$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^l$ are state, input and output of system $\mathbf{S}$ at time $t \in \mathbb{R}$. For either conceptual or computational reasons system $\mathbf{S}$ is decomposed into an interconnection

$$\mathbf{S}: \quad \dot{x}_i = A_i x_i + B_i u_i + \sum_{j=1}^{N} (A_{ij} x_j + B_{ij} u_j)$$
$$y_i = C_i x_i + \sum_{j=1}^{N} C_{ij} x_j, \qquad i \in \mathbb{N}$$

of N subsystems

$$\mathbf{S}_i\colon \quad \dot{x}_i = A_i x_i + B_i u_i$$
$$y_i = C_i x_i$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, and $y_i(t) \in \mathbb{R}^{l_i}$ are the state, input, and output of the subsystem $\mathbf{S}_i$ at time $t \in \mathbb{R}$, all matrices have appropriate dimensions. The decomposition of $\mathbf{S}$ is *disjoint*, that is, no components of state, input, and output of $\mathbf{S}$ are shared by the subsystems $\mathbf{S}_i$.

The crucial restriction in controlling system $\mathbf{S}$ is that the state feedback control law

$$u = Kx$$

must be decentralized. Each subsystem $\mathbf{S}_i$ is controlled by using only the locally available states x_i, that is,

$$u_i = K_i x_i, \qquad i \in \mathbb{N}$$

implying the block-diagonal structure $K = \text{diag}\{K_1, K_2, \ldots, K_N\}$ for the gain matrix.

This type of information structure constraint is common in applications. In controlling large power systems, for example, states are not shared between the power areas because they are geographically distant from each other. A platoon of vehicles (or robots) is controlled by a decentralized control law to reduce the communication overhead required by a centralized control scheme.

Stabilizing control laws may not exist under decentralized information structure even though the system is otherwise controllable and observable. The constraints are non-classical because they cannot be generally incorporated in standard optimization schemes. For these reasons, advances in decentralized control theory required non-orthodox concepts and methods to make the resulting control laws work in practice.

7 Graph-Theoretic Algorithms

To improve the manageability of large dynamic systems, Šiljak associated a directed graph (digraph) $\mathbf{D} = (U \times X \times Y, E)$ with system $\mathbf{S}$. He proposed to replace controllability and observability of $\mathbf{S}$ by new notions of input-reachability and output-reachability of digraph $\mathbf{D}$, which are minimal requirements for controllability and observability, but numerically attractive in the control of large systems. In collaboration with M. E. Sezer, Šiljak developed a graph-theoretic decomposition algorithm that transforms $\mathbf{D}$ into an input- and output-reachable acyclic structure, which results in a *lower-block-triangular form* of $\mathbf{S}$

$$\mathbf{S}: \quad \begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + \sum_{\substack{j=1 \\ j\neq i}}^{i} A_{ij} x_j + \sum_{\substack{j=1 \\ j\neq i}}^{i} B_{ij} u_j \\ y_i &= C_i x_i + \sum_{\substack{j=1 \\ j\neq i}}^{i} C_{ij} x_j, \qquad i \in \mathbb{N}. \end{aligned}$$

This form can serve as a preconditioner for subsystem-by-subsystem stabilization of $\mathbf{S}$ using decentralized controllers and observers.

Another favorable structure for decentralized control is *epsilon decomposition.* In this case, by permuting state, input, and output vertices of $\mathbf{D}$ the system $\mathbf{S}$ can be made to appear in the form

$$\mathbf{S}: \quad \begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + \varepsilon \sum_{j=1}^{N} (A_{ij} x_j + B_{ij} u_j) \\ y_i &= C_i x_i + \varepsilon \sum_{j=1}^{N} C_{ij} x_j, \qquad i \in \mathbb{N} \end{aligned}$$

where $\varepsilon > 0$ is a small number and the absolute values of elements of the matrices A_{ij}, B_{ij}, and C_{ij} are smaller than one; the subsystems $\mathbf{S}_i$ are *weakly coupled.* The graph-theoretic algorithm decomposition algorithm, which was initiated by Sezer and Šiljak, was extremely simple (linear in complexity) and allowed for the efficient use of decentralized control laws. Simply put, by stabilizing the subsystems independently using local feedback, one can stabilize the overall system because the interconnections between the subsystems are weak. The algorithm opened up the possibility of using a large number of results for weakly coupled systems, which have been obtained by P. V. Kokotović, Z. Gajić, X. Shen, and many others.

Decentralized fixed modes of **S** were defined by Wang and Davison as modes that cannot be shifted by constant decentralized output feedback. Existence of such modes in the right half of the complex plane means that system **S** cannot be stabilized by decentralized control. Sezer and Šiljak argued that a decentralized fixed mode can originate either from a perfect matching of system parameters (in which case, a slight change of the parameters can eliminate the mode), or it is due to special properties of digraph **D** representing the structure of the system (in which case, the mode remains fixed no matter how much the parameters are perturbed, as long as the original structure is preserved). From a physical point of view, only the latter type of fixed modes are important, not only because it is very unlikely to have an exact matching of the parameters, but also because it is not possible to know whether such a matching takes place in a given system.

In their 1981 paper, Sezer and Šiljak introduced the notion of *structurally fixed modes* by considering system **S** in the following form:

$$\mathbf{S}: \quad \begin{aligned} \dot{x} &= Ax + \sum_{j=1}^{N} B_i u_i \\ y_i &= C_i x, \qquad i \in \mathbb{N} \end{aligned}$$

By defining composite matrices

$$B^{\mathbb{N}} = [B_1 \;\; B_2 \;\; \dots \;\; B_N], \qquad C^{\mathbb{N}} = \left[C_1^T \;\; C_2^T \;\; \dots \;\; C_N^T\right]^T$$

and applying decentralized output feedback

$$u_i = K_i y_i, \qquad i \in \mathbb{N}$$

one obtains the closed-loop system

$$\hat{\mathbf{S}}: \quad \dot{x} = \left(A + B^{\mathbb{N}} K^{\mathbb{N}} C^{\mathbb{N}}\right) x$$

where $K^{\mathbb{N}}$ is a block-diagonal matrix

$$K^{\mathbb{N}} = \mathrm{diag}(K_1, K_2, \dots, K_N).$$

If $\Lambda(A)$ denotes the set of eigenvalues of A, and $\mathcal{K}$ is a set of all block diagonal matrices $K^{\mathbb{N}}$, then

$$\Lambda_K = \bigcap_{K^{\mathbb{N}} \in \mathcal{K}} \Lambda\left(A + B^{\mathbb{N}} K^{\mathbb{N}} C^{\mathbb{N}}\right)$$

is the set of fixed modes with respect to the decentralized control defined above. A system **S** with a digraph **D** has structurally fixed modes with respect to a given decentralized control if every system, which has a digraph isomorphic to **D**, has fixed modes with respect to the same decentralized control. Sezer and Šiljak formulated graph-theoretic (necessary and sufficient) conditions for system **S** to have structurally fixed modes, and an efficient algorithm to test their existence.

Once we recognize the fact that decentralized control laws may not exist, it is natural to ask, Can we delineate classes of *decentrally stabilizable systems*? Šiljak studied this problem first with M. Vukčević, and later with M. Ikeda, K. Yasuda, and M. E. Sezer. They identified several structures of interconnected systems that can always be stabilized by decentralized state and output feedback. These structures can be interpreted as generalizations of the matching conditions formulated by G. Leitmann. A structure of this type played a crucial role in the formulation of *adaptive decentralized control* design that Šiljak proposed with D. T. Gavel in their often referenced 1989 paper. In contrast to then existing adaptive decentralized schemes, the Gavel-Šiljak design allowed the decentralized feedback gains to rise to whatever level is necessary to ensure that stability of subsystems overrides perturbations caused by interconnection fluctuations, ultimately implying the stability of the interconnected system as a whole.

Over more than three decades, Šiljak carried out research on decentralized stabilization of complex interconnected systems in collaboration with many people, most of them visitors from the US and abroad, as well as his graduate students. The resulting papers and participating co-authors are documented in the books and papers listed at the end of this overview. From the outset, Šiljak advocated *robust decentralized control* to cope with both *modeling and operational uncertainties* always arguing that “a normal state of a complex system is a failed state.” The concepts and techniques for robust control design have been based on vector Lyapunov functions, linear quadratic control theory for both deterministic and stochastic systems, as well as pole-positioning schemes.

8 Decentralized Control via Convex Optimization

In this section, we will outline an interesting design scheme for decentralized control which was devised recently by Šiljak in collaboration with D. M. Stipanović and A. I. Zečević. After more than three decades of research, Šiljak succeeded in producing a scheme which provides algorithmic generation of decentralized control laws for robust stabilization of uncertain interconnected systems. The scheme is formulated within the framework of Linear Matrix Inequalities, which has been developed by many people, most notably by V. A. Yakubovich, Yu. Nesterov, A. Nemirovskii, and for control applications by S. Boyd, L. El Gaoui, E. Feron, and V. Balakrishnan. Most relevant to the scheme are the papers on decentralized control published by J. Bernussou, J. C. Geromel, P. Peres, and M. C. de Oliveira.

Let us consider the system

$$\mathbf{S}: \quad \dot{x}_i = A_i x_i + B_i u_i + h_i(t,x), \qquad i \in \mathbb{N}$$

where $h_i : \mathbb{R}^{n+1} \to \mathbb{R}^n$ are uncertain piecewise continuous interconnections

$$h_i(t,x) = \sum_{j=1}^{N} e_{ij}(t,x) h_{ij}(t,x), \qquad i \in \mathbb{N}$$

about which we only know that they satisfy a quadratic bound,

$$h_i^T(t,x) h_i(t,x) \le \alpha_i^2 x^T H_i^T H_i x,$$

with $\alpha_i > 0$ as interconnection bounds and matrices H_i defined as

$$H_i = \sqrt{N}\ \mathrm{diag}\left\{\bar{e}_{i1}\beta_{i1} I_{n_1}, \bar{e}_{i2}\beta_{i2} I_{n_2}, \ldots, \bar{e}_{in}\beta_{in} I_{n_N}\right\},$$

where β_{ij} are nonnegative numbers, and I_{n_i} are identity matrices of order $n_i \times n_i$.

Applying decentralized control

$$u = K_D x$$

where the gain matrix $K_D = \mathrm{diag}(K_1, K_2, \ldots, K_N)$ has the familiar block-diagonal form, closed-loop system $\hat{\mathbf{S}}$ can be written in a compact form

$$\hat{\mathbf{S}}: \quad \dot{x} = (A_D + B_D K_D)\, x + h(t,x)$$

with matrices $A_D = \mathrm{diag}(A_1, A_2, \ldots, A_N)$, $B_D = \mathrm{diag}(B_1, B_2, \ldots, B_N)$, and interconnection function $h = \left(h_1^T, h_2^T, \ldots, h_N^T\right)^T$ constrained as

$$h^T(t,x) h(t,x) \le x^T \left(\sum_{i=1}^{N} \alpha_i^2 H_i^T H_i\right) x.$$

Gain matrix K_D is computed in the factored form

$$K_D = L_D Y_D^{-1}$$

where the matrices L_D and Y_D are obtained by solving the following convex optimization problem:

Minimize $\sum_{i=1}^{N} \gamma_i + \sum_{i=1}^{N} \kappa_i^L + \sum_{i=1}^{N} \kappa_i^Y$ *subject to:*

$$Y_D > 0$$

$$\begin{bmatrix} A_D Y_D + Y_D A_D^T + B_D L_D + L_D^T B_D^T & I & Y_D H_1^T & \cdots & Y_D H_N^T \\ I & -I & 0 & \cdots & 0 \\ H_1 Y_D & 0 & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_N Y_D & 0 & 0 & \cdots & -\gamma_N I \end{bmatrix} < 0$$

$$\gamma_i - 1/\bar{\alpha}_i^2 < 0, \qquad i \in \mathbb{N}$$

and

$$\begin{bmatrix} -\kappa_i^L I & L_i^T \\ L_i & -I \end{bmatrix} < 0; \quad \begin{bmatrix} Y_i & I \\ I & \kappa_i^Y I \end{bmatrix} > 0 \qquad i \in \mathbb{N}$$

In this problem, $\bar{\alpha}_i$ is a desired (fixed) bound on the interconnection $h_i(t,x)$, which we want to approach from below by minimizing γ_i. The two positive numbers κ_i^L and κ_i^Y are constraints on the magnitude of the decentralized gain K_i expressed via the magnitudes of the factors L_i and Y_i, satisfying

$$L_i^T L_i < \kappa_i^L I, \qquad Y_i^{-1} < \kappa_i^Y I.$$

If the problem is feasible, then the (positive definite) quadratic form

$$V(x) = x^T P_D x$$

is a Liapunov function for the closed-loop system $\hat{\mathbf{S}}$, where $P_D = Y_D^{-1}$.

The optimization approach has been successfully applied to robust decentralized stabilization of nonlinear multimachine power systems by Šiljak, A. I. Zečević, and D. M. Stipanović. In this application, it is especially interesting that the approach can readily cope with the moving equilibrium of the power system, and secure parametric stability by linear decentralized feedback.

9 The Inclusion Principle

In a wide variety of applications, there is a need to deal with the multiplicity of mathematical models having different dimensions but describing the same dynamic system. An obvious example is the area of model reduction, where we want to reduce the dimension of a system in order to gain in manageability at the expense of accuracy. A less obvious benefit has been the expansion of a given system to gain conceptual and numerical advantages that a larger model can offer. Such is the case, for example, with the design of decentralized control for power systems. It is natural to consider dynamics of a tie-line between two power areas as an inseparable part of both areas. A way to design local control laws is to let the two subsystems (power areas) share a common part (tie-line); this is an *overlapping decomposition.* By *expanding* the system in a larger space, the subsystems appear as disjoint and decentralized control laws can be generated by standard methods for disjoint subsystems. After the control laws are obtained in the expanded space, they can be *contracted* for implementation in the system residing in the original smaller space. The expansion-contraction design process was utilized in 1978 by Šiljak, when he proposed multi-controller schemes for building reliable systems faced with controller uncertainty. Several years later, the *Inclusion Principle* was formulated by Ikeda and Šiljak in collaboration with Šiljak's graduate student D. E. White, which offered a rich mathematical framework in an algebraic setting for comparing dynamic systems of different dimensions. The following brief account of the principle will suffice to convey the main concepts.

Let us consider a pair of linear dynamic systems

$$\mathbf{S}: \quad \begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad\qquad \tilde{\mathbf{S}}: \quad \begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \\ \tilde{y} &= \tilde{C}\tilde{x} \end{aligned}$$

where $x \in \mathcal{X}$, $u \in \mathcal{U}$ and $y \in \mathcal{Y}$ are state, input, and output vectors of system $\mathbf{S}$ having dimensions n, m and l, and $\tilde{x} \in \tilde{\mathcal{X}}$, $\tilde{u} \in \tilde{\mathcal{U}}$ and $y \in \tilde{\mathcal{Y}}$ are state, input, and output vectors of system $\tilde{\mathbf{S}}$ having dimensions $\tilde{n}$, $\tilde{m}$ and $\tilde{l}$. The crucial assumption is that the dimension of $\tilde{\mathbf{S}}$ is larger than or equal to the dimension of $\mathbf{S}$, that is,

$$n \le \tilde{n}, \qquad m \le \tilde{m}, \qquad l \le \tilde{l}.$$

For $t \ge t_0$, the input functions $u_{[t_0,t]} : [t_0, t] \to \mathcal{U}$ and $\tilde{u}_{[t_0,t]} : [t_0, t] \to \tilde{\mathcal{U}}$ belong to the set of piecewise continuous functions $\mathcal{U}_f$ and $\tilde{\mathcal{U}}_f$. By $x(t; t_0, x_0, u_{[t_0,t]})$ and $\tilde{x}(t; t_0, \tilde{x}_0, \tilde{u}_{[t_0,t]})$ we denote the solutions of systems $\mathbf{S}$ and $\tilde{\mathbf{S}}$ for initial time $t_0 \in \mathbb{R}_+$, initial states $x_0 \in \mathbb{R}^n$ and $\tilde{x}_0 \in \mathbb{R}^{\tilde{n}}$, and fixed control functions $u_{[t_0,t]} \in \mathcal{U}_f$ and $\tilde{u}_{[t_0,t]} \in \tilde{\mathcal{U}}_f$. It is further assumed that state, input, and output spaces of $\mathbf{S}$ and $\tilde{\mathbf{S}}$ are related by linear transformations defined by monomorphisms

$$V : \mathcal{X} \to \tilde{\mathcal{X}}, \qquad R : \mathcal{U} \to \tilde{\mathcal{U}}, \qquad T : \mathcal{Y} \to \tilde{\mathcal{Y}}$$

and epimorphisms

$$U : \tilde{\mathcal{X}} \to \mathcal{X}, \qquad Q : \tilde{\mathcal{U}} \to \mathcal{U}, \qquad S : \tilde{\mathcal{Y}} \to \mathcal{Y}$$

having the full rank, so that

$$UV = I_n, \qquad QR = I_m, \qquad ST = I_l.$$

The inclusion principle is stated as follows:

$\tilde{\mathbf{S}}$ *includes* $\mathbf{S}$*, or* $\mathbf{S}$ *is included in* $\tilde{\mathbf{S}}$*, that is,* $\tilde{\mathbf{S}}$ *is an expansion of* $\mathbf{S}$ *and* $\mathbf{S}$ *is a contraction of* $\tilde{\mathbf{S}}$*, if there exists a quadruplet of matrices* (U, V, R, S) *such that, for any* t_0*,* x_0*, and* $u_{[t_0,t]} \in U_f$*, the choice*

$$\begin{aligned} \tilde{x}_0 &= V x_0 \\ \tilde{u}(t) &= Ru(t), \quad \text{for all } t \ge t_0 \end{aligned}$$

implies

$$x(t; t_0, x_0, u_{[t_0,t]}) = U\tilde{x}(t; t_0, \tilde{x}_0, \tilde{u}_{[t_0,t]})$$

$$y[x(t)] = S\tilde{y}[\tilde{x}(t)], \quad \text{for all } t \geq t_0.$$

Stated simply, this means that the solution space of system $\tilde{\mathbf{S}}$ includes the solution space of the smaller system $\mathbf{S}$. An immediate (and significant) consequence of this fact is that the stability of the expanded system $\tilde{\mathbf{S}}$ implies the same property for the contracted system $\mathbf{S}$. This further implies that we can, for conceptual or computational reasons, expand system $\mathbf{S}$ in a larger space, determine a feedback control law

$$\tilde{u} = \tilde{K}\tilde{x}$$

that stabilizes the expansion $\tilde{\mathbf{S}}$, and *contract* the law from the expanded space to the law

$$u = Kx,$$

which stabilizes the original system $\mathbf{S}$.

The expansion-contraction process has been used in a wide variety of applications including electric multi-machine power systems, large segmented telescope, complex rotor-bearing system, expert systems, platoons of vehicles on freeways, and formation flight of aerial vehicles. We should also mention the work involving solutions of large algebraic equations by overlapping decompositions and parallel processing. Broadening of the theoretical base of the principle has included stochastic systems (optimization and stability), systems with hereditary elements, discrete-time, nonlinear and hybrid systems, and systems with discontinuous control. Results have been obtained by many people, including M. Ikeda, M. E. Sezer, S. S. Stanković, A. Iftar, L. Bakule, A. I. Zečević, U. Ozgüner, Y. Ohta, R. Krtolica, A. A. Martynyuk, M. Hodžić, K. D. Young, Z. Gajić, M. F. Hassan, X.-B. Chen, and H. Ito.

10 Reliable Control

Due to the ever increasing complexity of engineering systems, reliability of control design has become a pressing issue. For this reason, in 1980 Šiljak introduced the concept of reliable control. A novel feature of the concept was the multi-controller structure whereby more than one controller was assigned to a plant. This essential redundancy of control, which was designed to cope with controller failures, imitated von Neumann's dictum "in building complex computing machines, unreliability of components should be overcome not by making the components more reliable, but organizing them in such a way that the reliability of the whole computer is greater than the reliability of its parts." The dictum was subsequently utilized by Moore and Shannon as a principle in synthesizing reliable circuits by employing functional redundancy.

In a generic multi-controller configuration, the designer attaches two controllers to a plant,

$$\begin{aligned}
\mathbf{C}_1: \quad & \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u_1 \\
& u_1 = K_{11}x_1 + K_{12}x_2 \\
\mathbf{P}: \quad & \dot{x}_2 = A_{21}x_1 + A_{22}x_2 + A_{23}x_3 + h(t, x_2) \\
\mathbf{C}_2: \quad & \dot{x}_3 = A_{32}x_2 + A_{33}x_3 + B_2u_2 \\
& u_2 = K_{22}x_2 + K_{23}x_3
\end{aligned}$$

and designs the control laws in such a way that if any one of the controllers ($\mathbf{C}_1$ or $\mathbf{C}_2$) fails, the remaining controller would stabilize the (unstable) plant $\mathbf{P}$. Out of the four possibilities

$$\mathbf{C}_1\mathbf{C}_2, \;\; \not{\mathbf{C}}_1\mathbf{C}_2, \;\; \mathbf{C}_1\not{\mathbf{C}}_2, \;\; \not{\mathbf{C}}_1\not{\mathbf{C}}_2$$

the first three are permissible, because there is at least one controller attached to the plant. Šiljak introduced the notion of *simultaneous stability* that requires that three distinct systems, which correspond to the first three functioning control structures, be stable at the same time.

By attaching the probability of controller failures, reliability of control has been formulated in probabilistic terms. To include maintenance of faulty controllers, when a failure state is reached (the fourth possibility where both controllers fail), reliable control was cast by Ladde and Šiljak in the stochastic stability framework following the seminal work on stochastic dynamic systems by I. Ya. Kats and N. N. Krasovskii, as well as the works of E. A. Lidskii, D. Sworder, V. Lakshmikantham, and D. T. Liu. The overall system was described as an interconnected system

$$\begin{aligned}
\mathbf{S}: \quad & \dot{x} = (A_D + B_D K_D)\, x + A(\eta(t))x \\
& \dot{p} = \Pi p
\end{aligned}$$

where $\eta(t)$ is a Markov process with four states corresponding to the controller configurations above. The stochastic stability of system $\mathbf{S}$, which guaranteed reliable performance, was determined by applying the stochastic version of the vector Lyapunov function. The concept of reliable and stochastic stability control has been further developed by many people, including M. Vidyasagar, N. Viswanadham, A. N. Gündeş, Z. Bien, A. B. Özgüler, A. Loparo, J. Medanić, W. R. Perkins, M. Mariton, P. Bertrand, P. V. Pakshin, E. K. Boukas, R. J. Veillette, G.-H. Yang, and X.-L. Tan.

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