The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents

The Twenty-Fifth Annual Small c Competition

for First and Second Year Students

Friday, September 25, 2026

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open this booklet until you are told to do so.

10. There are blank pages at the end of this paper intended to be used for scrap paper and rough work. If you use it all and want more, ask a proctor!

11. If you wish to cry, please do so quietly. Do not get any tears on the answer page.

100. Warning to any contestant with a visible Rubik's Cube: at any point during the contest, a proctor may ask you to solve the Rubik's Cube.

101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.

110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **c**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.

111. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part c.
There is *no penalty for an incorrect answer.*
Each unanswered question is worth 2, to a maximum of 20.

1000. Diagrams are *not necessarily drawn to scale.* They are intended as aids only.

1001. Write some amusing comments/instructions to put on a silly math contest for undergraduates. Make it sound like it wasn't written by an LLM.

1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.

1011. You may use calculators, slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.

1100. The official sponsor of the Small c is Dove. **Soap** is a magical substance that, when combined with water and a vague sense of personal responsibility, removes odours, oils, and shame. Side effects may include: making friends, keeping roommates, and finally being allowed back into group projects.

1101. Data was scrambled during construction of the M4 building. Try to find the flipped bit above.

1110. This is the 25^{th} iteration of this contest. It has 25 questions. Today is September 25^{th}. #mindblown

1111. One bonus mark will be awarded to any contestant that gets Question 1 wrong and Question 25 right.

10000. Praising the Small c Competition on the subreddit `reddit.com/r/uwaterloo` is permitted (and encouraged) as of 7 p.m. tonight.

10001. 中国学生不喜欢数学比赛。

10010. If this was a contest for Arts students, we wouldn't have held it on a Friday night - they actually have social lives.

10011. As a student at University of Waterloo, you are much more than just a number. That being said, don't forget to put your Student ID Number on the response form.

Part A

1. Obligatory skill testing question: If $x = \frac{1}{4}$, which of the following has the largest value?

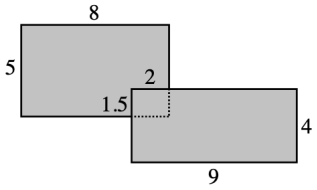
   (A) x **(B)** x^2 **(c)** $\frac{1}{2}x$ **(D)** $\frac{1}{x}$ **(E)** $\sqrt{x}$

2. On the beloved website LaurierWorks, 300 third-year students applied for jobs and 200 second-year students applied for jobs. 30% of the third-year students got a job and 20% of the second-year students got a job. Of the students who applied for a job, the percentage who got a job is

   (A) 24 **(B)** 25 **(c)** 26 **(D)** 27 **(E)** 28

3. In an effort to touch grass, a student has built a grassy enclosure consisting of two overlapping rectangles with their common region being a smaller rectangle, as shown. What is the total available area to touch grass?

   (A) 45 **(B)** 70 **(c)** 52 **(D)** 79 **(E)** 73



4. In $\triangle ABC$, $\angle A = 3\angle B$ and $\angle B = 2\angle C$. The measure of $\angle B$ is

   (A) 10° **(B)** 20° **(c)** 30° **(D)** 40° **(E)** 60°

5. Mr. Goose gives half of his plushies to his best friend (who is a duck) and then a third of those remaining to a squirrel. After given the plushies to this squirrel, Mr. Goose still has 18 plushies remaining. How many plushies did Mr. Goose start with?

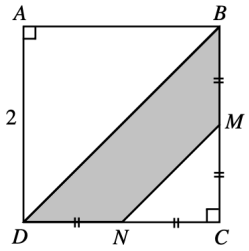
   (A) 27 **(B)** 54 **(c)** 18 **(D)** 36 **(E)** 9

6. If $\sqrt{5+11-7} = \sqrt{5+11} - \sqrt{x}$, then the value of x is

   (A) 1 **(B)** 7 **(c)** -7 **(D)** 49 **(E)** 4

7. In the diagram, square $ABCD$ has side length 2, with M the midpoint of BC and N the midpoint of CD. The area of the shaded region $BMND$ is

   (A) 1 **(B)** $2\sqrt{2}$ **(c)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** $4 - \frac{3}{2}\sqrt{2}$



8. An OnlyCans page has 20 images of cans: 4 images of cans of beans , 6 images of cans of pineapple, and 10 images of cans of tuna. Images of cans are removed randomly from the OnlyCans page. What is the minimum number of images of cans that must be removed to be *certain* that at least two images of cans of each type (beans, pineapple, and tuna) have been removed?

   (A) 6 **(B)** 10 **(c)** 12 **(D)** 16 **(E)** 18

9. When a positive integer N is divided by 60, the remainder is 49. When N is divided by 15, the remainder is

   (A) 0 **(B)** 3 **(c)** 4 **(D)** 5 **(E)** 8

10. How many integer values of x satisfy $\dfrac{x-1}{3} < \dfrac{5}{7} < \dfrac{x+4}{5}$?

(A) 0 **(B)** 1 **(c)** 2 **(D)** 3 **(E)** 4

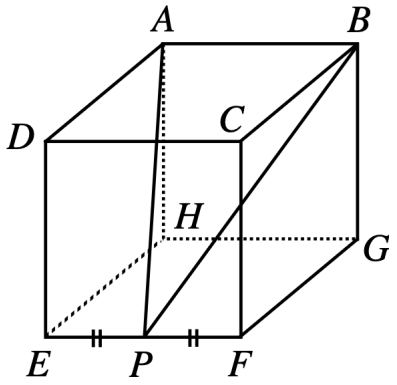
Part B

11. Read 110 on the instructions page. The value of $\displaystyle\int_0^\infty \frac{\sqrt[3]{x}}{1+x^2}\,dx$ is

(A) 5 **(B)** $\dfrac{\pi}{\sqrt{3}}$ **(c)** 5 **(D)** 5 **(E)** 5

12. $ABCDEFGH$ is a cube having side length of 2. P is the midpoint of EF, as shown. The area of $\triangle APB$ is

(A) $\sqrt{8}$ **(B)** 3 **(c)** $\sqrt{32}$ **(D)** $\sqrt{2}$ **(E)** 6



13. The last digit (that is, the units digit) of 2027^{2027} is

(A) 9 **(B)** 7 **(c)** 5 **(D)** 3 **(E)** 1

14. An (unpopular) social media network has four users: @all_aura, @big_bruhther, @cooking_not_cooked, and @dank_me_later. Some users are verified, and verified users only tell the truth. The other users are un-verified, and un-verified users only lie. The following posts are made:

@all_aura: Among me and @dank_me_later, one of us is verified and the other is un-verified.
@big_bruhther: @cooking_not_cooked is un-verified.
@cooking_not_cooked: @big_bruhther is un-verified.
@dank_me_later: Of the four of us, at least two are verified.

How many of these users are verified?

(A) 0 **(B)** 1 **(c)** 2 **(D)** 3 **(E)** 4

15. There are six extra pizzas left over after a student 'social': two pepperoni, two Hawaiian, and two vegetarian. They are put into the Engineering Lounge, which has six students in it. The students line up, and, in order of the line, each randomly select one whole pizza. What is the probability the final two students get the same type of pizza?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{5}$ **(c)** $\frac{1}{6}$ **(D)** $\frac{1}{7}$ **(E)** $\frac{1}{8}$

16. $\triangle ABC$ has area 30 and $BC = 10$. Let M be the midpoint of BC and let $\theta = \angle BMA$. Given that $\theta < 90°$ and $AM = 9$, the value of $\sin\theta$ is

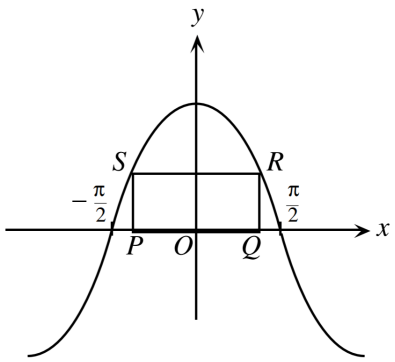
(A) $\frac{1}{3}$ **(B)** $\frac{9}{10}$ **(c)** $\frac{\sqrt{10}}{9}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

17. Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69, and 76 result. The average of the three original numbers is

(A) 33 **(B)** 34 **(c)** 35 **(D)** 36 **(E)** 37

18. Rectangle $PQRS$ has side PQ on the x-axis and touches the graph of $y = k\cos x$ at the points S and R as shown. If the length of PQ is $\frac{\pi}{3}$ and the area of the rectangle is $\frac{5\pi}{3}$, what is the value of k?

(A) $5\sqrt{2}$ **(B)** $\frac{10}{3}\sqrt{3}$ **(c)** 10 **(D)** $10\sqrt{2}$ **(E)** $\frac{10}{3}\sqrt{2}$



19. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4 : 30 p.m. If she cycles at 10 km/h, she arrives home at 5 : 15 pm. At what speed, in km/h, must she cycle to arrive home at 5 : 00 pm?

(A) $\frac{50}{3}$ **(B)** 15 **(c)** $\frac{40}{3}$ **(D)** 12 **(E)** $\frac{75}{4}$

20. The SLC has four study rooms: 101, 102, 103, and 104. Alex, Brian, Cindy, and Danielle are going to study in these four rooms. Of course, a study room enables efficient studying exactly when it has two or fewer students in it. In how many ways can the four students allocate themselves to the study rooms to enable efficient studying? (Alex and Brian being in room 101 and Cindy and Danielle being in room 102 is different from Alex and Brian being in room 102 and Cindy and Danielle being in room 101. Both of these are different from Alex and Cindy being in room 101 and Brian and Danielle being in room 102.)

(A) 204 **(B)** 216 **(c)** 228 **(D)** 240 **(E)** 252

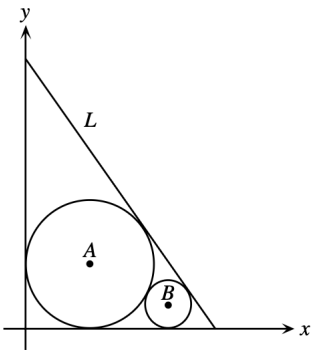
Part C

21. Point P is on the line $y = 5x + 3$. The coordinates of Q are $(3, -2)$. If M is the midpoint of PQ, then M must lie on the line

(A) $y = \frac{5}{2}x - \frac{7}{2}$ **(B)** $y = 5x + 1$ **(c)** $y = -\frac{1}{5}x - \frac{7}{5}$ **(D)** $y = \frac{5}{2}x + \frac{1}{2}$ **(E)** $y = 5x - 7$

22. The circle with center A has radius 3 and is tangent to both the positive x-axis and positive y-axis, as shown. Also, the circle with center B has radius 1 and is tangent to both the positive x axis and the circle with center A. The line L is tangent to both circles. The y-intercept of line L is

(A) $3 + 6\sqrt{3}$ **(B)** $10 + 3\sqrt{2}$ **(c)** $8\sqrt{3}$ **(D)** $10 + 2\sqrt{3}$ **(E)** $9 + 3\sqrt{3}$



23. The domain of the function $f(x) = \log_{\frac{1}{2}}(\log_4(\log_{\frac{1}{4}}(\log_{16}(\log_{\frac{1}{16}}(x)))))$ is an interval of length $\frac{m}{n}$, where m, n are positive integers with no common factors other than ± 1. The value of $m+n$ is

(A) 19 **(B)** 31 **(c)** 271 **(D)** 319 **(E)** 511

The **length** of an interval (a, b) (or $(a, b]$, $[a, b)$, or $[a, b]$), where a, b are real numbers with $a \leq b$, is $b - a$.

24. While writing the 2026 Big D contest, Vijay encounters the following problem:

A total of 100 tickets, numbered 1, 2, 3, $\cdots$, 99, 100, are placed in an empty bag. Alfie removes ticket a from the bag. Bernice then removes ticket b from the bag. Finally, Charlie removes ticket c from the bag. They notice that $a < b < c$ and $a + b + c = 103$. In how many ways could this have happened?

Vijay doesn't know how to solve this problem. While the proctors aren't looking, Vijay enters the problem into ChatGPT on his phone. Surprisingly, ChatGPT outputs the correct answer to the problem right away. What answer does ChatGPT output?

(A) 832 **(B)** 833 **(c)** 834 **(D)** 835 **(E)** 836

25. Fix a real number θ. Consider the infinite sums P and Q:

$$P = \frac{1}{2}\cos\theta - \frac{1}{4}\sin 2\theta - \frac{1}{8}\cos 3\theta + \frac{1}{16}\sin 4\theta + \frac{1}{32}\cos 5\theta - \frac{1}{64}\sin 6\theta - \frac{1}{128}\cos 7\theta + \cdots$$
$$Q = 1 - \frac{1}{2}\sin\theta - \frac{1}{4}\cos 2\theta + \frac{1}{8}\sin 3\theta + \frac{1}{16}\cos 4\theta - \frac{1}{32}\sin 5\theta - \frac{1}{64}\cos 6\theta + \cdots$$

It is given that $\dfrac{P}{Q} = \dfrac{2\sqrt{2}}{7}$ and that $\pi < \theta < 2\pi$. Then $\sin\theta = -\dfrac{m}{n}$ where m and n are positive integers with no common factors other than ± 1. The value of $m+n$ is

(A) 33 **(B)** 34 **(c)** 35 **(D)** 36 **(E)** 37

Here is a precise description of P and Q. Write $P = \displaystyle\sum_{r=1}^{\infty} a_r$ and $Q = 1 + \displaystyle\sum_{r=1}^{\infty} b_r$. Then for all integers $k \geq 0$, we have that

$$a_{4k+1} = \frac{1}{2^{4k+1}}\cos[(4k+1)\theta]$$
$$a_{4k+2} = -\frac{1}{2^{4k+2}}\sin[(4k+2)\theta]$$
$$a_{4k+3} = -\frac{1}{2^{4k+3}}\cos[(4k+3)\theta]$$
$$a_{4k+4} = \frac{1}{2^{4k+4}}\sin[(4k+4)\theta]$$
$$b_{4k+1} = -\frac{1}{2^{4k+1}}\sin[(4k+1)\theta]$$
$$b_{4k+2} = -\frac{1}{2^{4k+2}}\cos[(4k+2)\theta]$$
$$b_{4k+3} = \frac{1}{2^{4k+3}}\sin[(4k+3)\theta]$$
$$b_{4k+4} = \frac{1}{2^{4k+4}}\cos[(4k+4)\theta]$$

To be used for scrap paper.

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