

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents*

The Twenty-First Annual Small c Competition

for First and Second Year Students

Monday 26 September 2022

Time: 1 hour

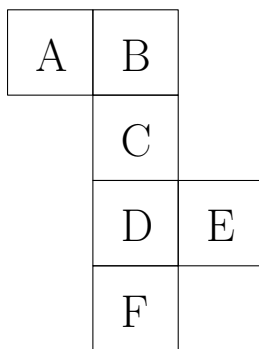
Calculators are permitted.

Instructions:

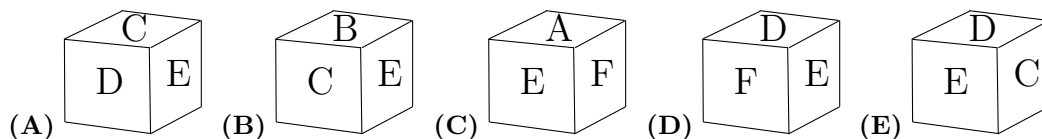
1. Do not open this booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.
110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, the dumb humans have been replaced by even dumber machines.
1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty for an incorrect answer*.
Each unanswered question is worth 2, to a maximum of 20.
1001. Diagrams are *not necessarily drawn to scale*. They are intended as aids only.
1010. Als u dit kunt lezen, spreekt u het Nederlands.
1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.
1100. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
1001. The only website you may use during the contest is **theonion.com**.
1110. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
1111. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.
10000. Hint: The answer to at least one question is **B**.
10001. One bonus mark will be awarded to any contestant that gets Question 1 wrong and Question 25 right.
10010. Praising the Small c Competition on the subreddit **reddit.com/r/uwaterloo** is permitted as of 7 p.m. tonight.
10011. Due to an unfortunate virus, the 2020 edition of the Small c contest was deleted. If you recover this contest, please contact the CEMC immediately.
10100. Don't forget to turn off the Water(loo) tap before you leave the Kitchen(er).

Part A

- The team of MathVengers began with 6 members. Through the course of their adventures, 10 more members joined the team. After these members joined, in a tragic event known as the Singularity, half of the team disappeared. Following the Singularity, two new members joined the team before the final battle against the evil villain Theta. How many MathVengers team members were there when the final battle happened?
 (A) 6 (B) 8 (C) 9 (D) 10 (E) 18
- The mean (average) of the three numbers 5, x , 7 is equal to 7. What is the value of x ?
 (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
- How many positive integers less than 100 are divisible by all of 1, 2, 3, and 4?
 (A) 5 (B) 6 (C) 7 (D) 8 (E) 9
- If $x^y = 7$, then what is the value of x^{2y} ?
 (A) 2 (B) 9 (C) 14 (D) 28 (E) 49
- Of the 20 members of UWaterloo's official Small c team, one-fourth of the members live on campus and one-fifth of the members are not math majors. Three members of the team who live on campus are not math majors. How many members of the Small c team are math majors living off campus?
 (A) 14 (B) 15 (C) 16 (D) 11 (E) 17
- Azani is sitting on a playground spinner, which is a large rotating disc. She is 4 metres from its centre. If the disc makes 6 revolutions per minute, at what speed is Azani travelling, in metres per minute?
 (A) 8π (B) 16π (C) 24π (D) 48π (E) 96π
- The following net is laid out on a table:



This net is folded up to make a cube. Ignoring the orientation of the letters, which of the following cannot be a picture of the cube after it has been folded up?

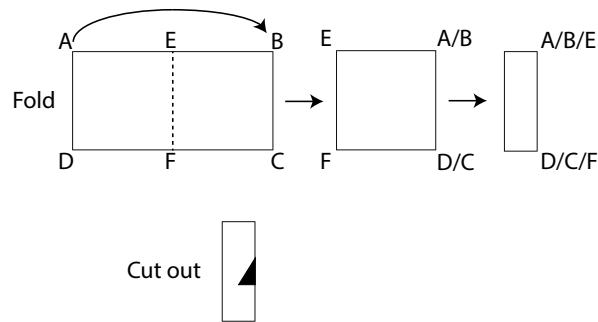


- What is the global minimum value attained by the function $f(x) = e^x - x$?
 (A) -1 (B) 0 (C) 1 (D) e (E) No global minimum value exists
- How many intersection points do the graphs of $f(x) = \sin x$ and $g(x) = \tan x$ have on the interval $0 \leq x \leq 2\pi$?
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

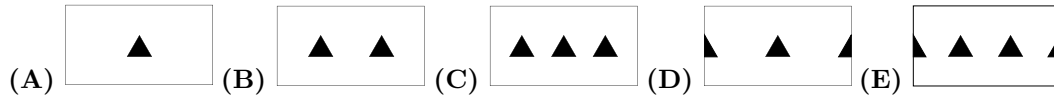
10. Quartus and Nonius have a fixed rotation for their school outfits. Quartus has 4 different outfits he wears to school, one each day, for four days. He then washes all four at once and the cycle starts over again. Nonius has 9 different outfits she wears to school, one each day, for nine days. She then washes all nine at once and the cycle starts over again. Both students attend school for 195 days. At the end of the 195-day school year, what is the difference between the number of clean outfits Nonius has left in her 9-outfit cycle and the number of clean outfits Quartus has left in his 4-outfit cycle?
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Part B

11. A rectangular sheet of paper is folded in half so that corner A is on corner B and corner D is on corner C. It is folded in half again so that E is on A/B and F is on D/C. A triangle is cut out as shown in the diagram.



When the paper is unfolded the paper will appear as



12. The variables x and y represent non-zero real numbers such that $xy = \frac{x}{y} = x - y$. What is the value of $x + y$?
- (A) -1 (B) $-\frac{3}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1
13. A Spray-O-Matic machine contains 256 mL of water. After the start button is pressed, the machine sprays water every 20 minutes, starting 20 minutes after the button press. The first spray uses 1 mL of water and each spray after that uses double the amount of the previous spray. In the case that not enough water is available, the spray just uses the amount of water left in the machine. If the machine instantaneously sprays the water, how many minutes after the start button is pressed will it take to completely empty the tank?
- (A) 220 (B) 200 (C) 180 (D) 160 (E) 140
14. One hundred students took UWaterloo's infamous MATH 999 course. The average mark of those who passed the course was 75%. The average mark of the other students was 35%. The average mark of all the students was 65%. How many students passed the course?
- (A) 75 (B) 25 (C) 80 (D) 70 (E) 65

15. Cards in a deck are coloured with red, blue, and/or green. Each card has at least one colour, and may have two or even all three colours on it. You know the following information about the cards in the deck:

- All cards that have red also have blue.
- At least one card has both blue and green on it.
- At least one card has green but does not have blue on it.

Which of the following statements must be true about the cards in the deck?

- (A) All cards with blue also have red
 (B) At least one card with red also has green on it
 (C) Not all cards with red on them have green on them
 (D) At least one card with green does not have red on it
 (E) At least one card with blue does not have red on it

16. An ambitious cyclist is travelling from their home city to another city for a cycling race. If they bike at an average speed of 20 km/h, they will reach their destination at exactly 5 p.m. Leaving at the same time, if they travel at an average speed of 30 km/h, they will reach their destination at exactly 3 p.m. Again, leaving at the same time, at what average speed must the cyclist travel to reach their destination by exactly 4 p.m.?

- (A) 24 km/h (B) 22 km/h (C) 23 km/h (D) 25 km/h (E) 26 km/h

17. What is the value of the 98-term product $(\log_2 3)(\log_3 4)(\log_4 5) \cdots (\log_{99} 100)$?

- (A) $\log_{10}(98)$ (B) $\log_{99!} \left(\frac{100!}{2!} \right)$ (C) $\log_2(100)$ (D) $\log_{4949}(5047)$ (E) $\ln(1)^{98}$

18. What is the mean (average) of all real numbers x that satisfy the equation

$$\frac{x^2 - 3x + 2}{3x^2 + 2x - 1} = \frac{x^2 - 3x + 2}{x^2 - x - 2}?$$

- (A) $\frac{1}{4}$ (B) $-\frac{1}{6}$ (C) $\frac{5}{6}$ (D) $-\frac{3}{4}$ (E) No values of x satisfy the equation

19. A sequence h_1, h_2, h_3, \dots is constructed so that h_n is the number of equilateral triangles with side length 1 that are needed to fill a regular hexagon with side length n . The following images show that $h_1 = 6$, and $h_2 = 24$. What is the value of h_4 ?



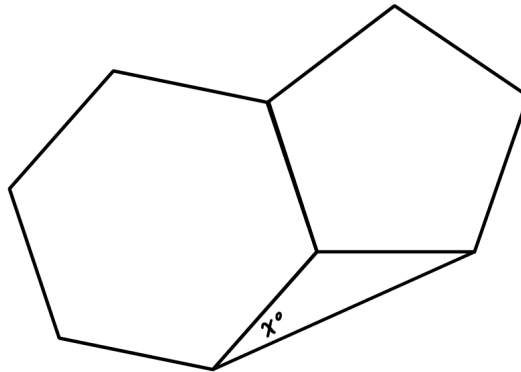
- (A) 54 (B) 60 (C) 96 (D) 150 (E) 384

20. How many two-digit positive integers AB with tens digit $A \neq 0$ are there such that the ones digit B divides AB ?

- (A) 36 (B) 39 (C) 38 (D) 41 (E) 40

Part C

21. Pictured below is a regular hexagon and regular pentagon, with the same side length. These two figures are joined at one of their sides, and a line segment is drawn between them as shown below. What is the value of x ?



- (A) 18 (B) 24 (C) 26 (D) 28 (E) 30
22. Given the list of natural numbers $1, 2, \dots, n$, one number m is removed from the list, and the mean of the remaining numbers is calculated to be $13\frac{9}{13}$. What is the value of the sum $m + n$?
- (A) 31 (B) 39 (C) 49 (D) 86 (E) 89
23. If $x + \frac{1}{x} = 3$, what is the value of $x^3 + \frac{1}{x^3}$?
- (A) 7 (B) 18 (C) 21 (D) 24 (E) 27
24. In celebration of Pi Day, 24 students in MathSoc get together for a party. Each of the 24 students selects exactly 15 of the other 23 people at the party, and gives each of them a pie. If two students give each other pies, they are called pie-pals. What is the minimum possible number of pairs of pie-pals?
- (A) 69 (B) 75 (C) 84 (D) 105 (E) 120
25. The product
- $$(2^6 - 1)(3^6 - 1)(4^6 - 1) \cdots (200^6 - 1)$$
- is evaluated and written as usual in base 10. How many zeroes are at the end of this number?
- (A) 39 (B) 42 (C) 79 (D) 95 (E) 96