

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents*

The Nineteenth Annual Small c Competition

for First and Second Year Students

Friday 27 September 2019

Time: 1 hour

Calculators are permitted.

Instructions:

1. Do not open this booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.
110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, the dumb humans have been replaced by even dumber machines.
1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty for an incorrect answer*.
Each unanswered question is worth 2, to a maximum of 20.
1001. Diagrams are *not necessarily drawn to scale*. They are intended as aids only.
1010. Als u dit kunt lezen, spreekt u het Nederlands.
1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.
1100. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
1001. The only website you may use during the contest is **theonion.com**.
1110. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
1111. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.
10000. Hint: The answer to at least one question is **B**.
10001. If you finish early, please Brexit the room quietly.
10010. Don't give up. As shown by the last U.S. presidential election, anyone can win this thing.
10011. One bonus mark will be awarded to any contestant that gets Question 1 wrong and Question 25 right.
10100. Praising the Small c Competition on the subreddit [reddit.com/r/uwaterloo](https://www.reddit.com/r/uwaterloo) is permitted as of 7 p.m. tonight.

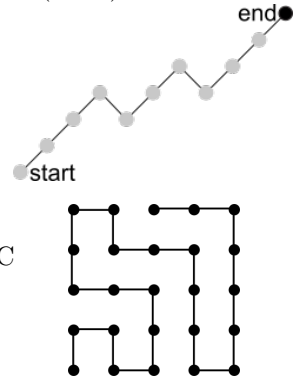
Part A

1. What is the value of $8 \div 2 \times (2 + 2)$? *Hint: Some would argue this is equivalent to $8 \div 2(2 + 2)$.*

- (A) 16 (B) 1 (C) 8 (D) 3 (E) 5

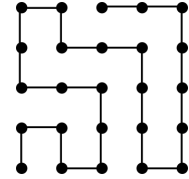
2. Dots are arranged on a path as shown. One of the grey dots is chosen at random. What is the probability that the next dot on the path is higher?

- (A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{7}{10}$ (D) $\frac{4}{5}$ (E) $\frac{9}{10}$



3. The 4 cm by 4 cm grid shown models the path taken by a student trying to avoid SLC construction. What is the total length of the path on the grid?

- (A) 25 cm (B) 24 cm (C) 26 cm (D) 27 cm (E) 23 cm



4. This question is in honour of James Holzhauer. Select the question that matches the answer.

Answer: 4

- (A) What is the value of $\log_{10^2} (10^9)$?
 (B) What is the value of $\log_{10^3} (10^{12})$?
 (C) What is the value of $\log_{10^5} (10^{15})$?
 (D) What is the value of $\log_{10^7} (10^{26})$?
 (E) What is the value of $\log_{10^4} (10^{14})$?

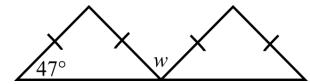
5. During a 21 minute show, 14000 fireworks were used. On average, how many fireworks per second were used?

Note that the answer to next year's Canada Day version of this question is not defined.

- (A) $\frac{55}{6}$ (B) $\frac{17}{3}$ (C) $\frac{42}{5}$ (D) $\frac{23}{4}$ (E) $\frac{100}{9}$

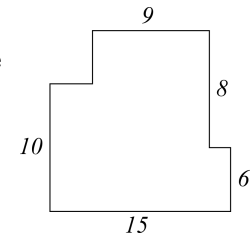
6. Congruent triangles lie on a straight line as shown. What is the measure of angle w ?

- (A) 86° (B) 90° (C) 47° (D) 133° (E) 88°



7. Lime scooters must remain within the figure shown. All angles are right angles and some lengths are indicated. What is the perimeter of the figure?

- (A) 50 (B) 52 (C) 54 (D) 56 (E) 58



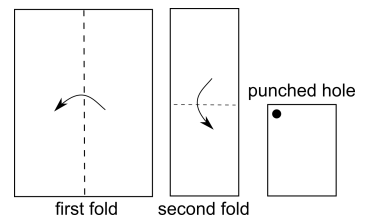
8. What is the maximum value of the expression $12x - 4x^2 - 5$?

- (A) -4 (B) 4 (C) 0 (D) -5 (E) 5

9. If 1% of used straws end up in the ocean and each of 8 billion people use one new straw every month, how many straws end up in the ocean over 5 years?

- (A) approximately 500000 (B) between 1 and 2 million (C) 4.8 million
 (D) between 400 and 500 million (E) over 4 billion

10. One page of the Medium D Contest is folded length wise and then folded width wise as shown. Then a hole is punched in the top left corner of the folded page as shown. This hole goes directly through the folded page making four holes in the original unfolded page. If the page is then unfolded without changing its orientation, which of the following shows the correct placement of holes?



- (A) (B) (C) (D) (E)

Philosophical distraction: At what point during the unfolding, does one hole turn into more than one hole?

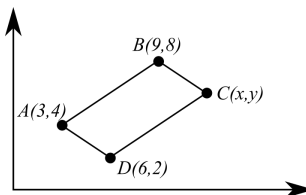
Part B

11. If $x = \frac{\pi}{4}$, what is the value of $\sin^4 x + 2 \sin^2 x \cos^2 x + \cos^4 x$?

- (A) 1 (B) 2 (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$ (E) $\frac{\sqrt{2}}{3}$

12. Parallelogram $ABCD$ is shown. What are the coordinates of the point C ?

- (A) (10, 10) (B) (12, 7) (C) (12, 6) (D) (15, 7) (E) (15, 6)



13. Derifun swims at an average of 1.25 m/sec and it takes 0.5 seconds for him to make a flip and turn at the wall. He enters swim races at a university down the road because they have a competition pool with lanes that are 50 m long. What is Derifun's average time for a 200 m long race?

- (A) 160.0 seconds (B) 160.5 seconds (C) 161.5 seconds
(D) 162.0 seconds (E) 162.5 seconds

14. What is the smallest perfect square divisible by 21 and 35?

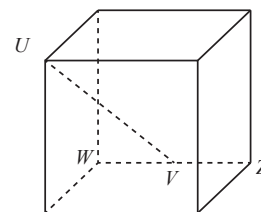
- (A) 105 (B) 735 (C) 1225 (D) 11025 (E) 540225

15. For how many ordered pairs of positive integers (x, y) is $x + 2y = 100$?

- (A) 33 (B) 49 (C) 50 (D) 67 (E) 98

16. The length of each edge in the cube shown is 1. Point V is the midpoint of edge WZ . What is the length of UV ?

- (A) 3 (B) 2 (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{\sqrt{6}}{2}$ (E) $\frac{3}{2}$



17. How many integers between 1000 and 2000 have the property that the sum of its first two digits is equal to the sum of its last two digits?

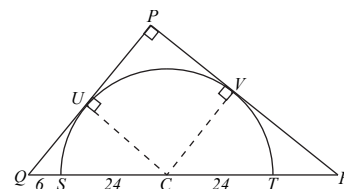
- (A) 63 (B) 64 (C) 65 (D) 66 (E) 67

18. First, note that this is not a CS course, so $0 \notin \mathbb{N}$. Now, suppose $\frac{a}{b} = \frac{2}{5}$, $\frac{b}{c} = \frac{15}{4}$, and $\frac{c}{d} = \frac{8}{13}$ for $a, b, c, d \in \mathbb{R}$. If $\frac{a+b+c}{d} = \frac{m}{n}$ for $m, n \in \mathbb{N}$ such that the greatest common divisor of m and n is 1, what is $m + n$?

- (A) 21 (B) 47 (C) 58 (D) 63 (E) 1499

19. A semicircle is tangent to two sides of right $\triangle PQR$ at U and V . Point C is the midpoint of the diameter ST which is divided into four segments as shown: $QS = 6$, $SC = 24$, $CT = 24$, and TR . What is the length of TR ?

- (A) 6 (B) 16 (C) 8 (D) 18 (E) 40



Philosophical distraction: In an earlier draft, we said Point C is the centre of a circle. It is presumably the centre of "the circle" but is it the centre of the semicircle?

20. Twelve friends with different names were randomly seated at a round table at the Bombshelter. If Alicia, Bao and Consuela were sitting in consecutive seats, what is the probability that Darpana and Ehsaan were not seated beside each other?

- (A) $\frac{7}{9}$ (B) $\frac{5}{6}$ (C) $\frac{5}{9}$ (D) $\frac{4}{9}$ (E) $\frac{1}{6}$

Part C

21. What is the sum of the digits of the integer $100^{17} - 17$?
- (A) 299 (B) 308 (C) 21 (D) 290 (E) 300
22. Mr. Goose is learning how to fly straight up and down at a constant velocity of 10 m/s. He starts on the ground, goes up 1 m and comes back down 0.5 m. Then he goes up 2 m and comes back down 1 m. Then he goes up 3 m and comes back down 1.5 m. Mr. Goose continues to go up one additional meter each time and then come back down half this vertical distance. He always changes direction instantaneously. For how many seconds will Mr. Goose have been flying when he reaches a height of 45 m for the first time?
- (A) 6.7 (B) 7.8 (C) 9.9 (D) 11.1 (E) 11.7
23. A teacher needs to place T identical tests on desks. There are D desks and they are arranged in a single row. No two tests can be placed on the same desk or on desks that are right beside each other. For which of the following values of T and D does the teacher have the largest number of ways to do this?
- (A) $D = 10$ and $T = 3$ (B) $D = 11$ and $T = 4$ (C) $D = 12$ and $T = 5$
(D) $D = 13$ and $T = 2$ (E) $D = 17$ and $T = 8$
24. How many different functions of the form $f(x) = x^3 + ax^2 + bx + c$ are there such that $f(r) = 0$ if and only if $r = a$, $r = b$ or $r = c$?
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
25. For how many positive integers N , is there an isosceles triangle with perimeter 2019 and area N ?
- (A) 196121 (B) 196122 (C) 196123 (D) 196124 (E) 196125

Philosophical distraction: If $N = 0$ were allowed, would the answer go up by one? And, no, zero is not positive!!!