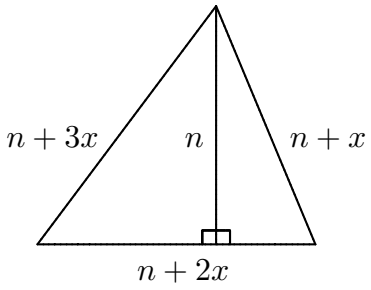
SPECIAL K
Saturday November 1, 2008
9:00 am - 12:00 noon

1: Find the value of $\min\limits_{|y|\le 1} \max\limits_{|x|\le 1}(x^2+xy)$ and the value of $\max\limits_{|x|\le 1} \min\limits_{|y|\le 1}(x^2+xy)$.

2: Let $f(x) = x^3 - 5x + 1$ and let $g(x) = \dfrac{x-1}{x^2}$. Find the number of x-intercepts on the graph of $y = f(g(x))$.

3: Determine the number of triangles which have the form shown below, where n is a positive integer and x is a real number with $0 \le x \le 1$.



4: A bathroom floor is tiled by regular hexagons of the same size. Points A, B and C are vertices of hexagons in the tiling. Prove that $\angle ABC \neq 45°$.

5: Show that the product $P(n) = \displaystyle\prod_{k=1}^{n-1} \frac{k^{2k}}{k^{n+1}}$ is an integer whenever n is prime.

6: Determine whether the sequence $\left\{\dfrac{1}{n\sin n}\right\}$ converges.

BIG E
Saturday November 1, 2008
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1: Let A be a 10×10 matrix with integer entries. Suppose that 92 of those entries yield a remainder of 1 after division by 3. Show that $\det(A)$ is a multiple of 3.

2: Let $f(x) = x^3 - 5x + 1$ and let $g(x) = \dfrac{x-1}{x^2}$. Find the number of x-intercepts on the graph of $y = f(g(x))$.

3: Find all twice differentiable functions $f(x)$ defined on $(0, \infty)$ such that $f'(x) > 0$ and $f\big(f'(x)\big) = -f(x)$ for all $x > 0$.

4: Show that the product $P(n) = \displaystyle\prod_{k=1}^{n-1} \frac{k^{2k}}{k^{n+1}}$ is an integer whenever n is prime.

5: Determine whether the sequence $\left\{ \dfrac{1}{n \sin n} \right\}$ converges.

6: Let $\pi > \alpha_1 > \alpha_2 > \alpha_3 > \cdots$ with each $\alpha_n > 0$. Form a polygonal path which winds clockwise starting with a line segment A_0A_1 of length 1, and adding edges A_nA_{n+1} all of length 1, with angle $A_{n-1}A_nA_{n+1}$ equal to α_n. Prove that there is a point contained in the interiors of all of these angles. (The *interior* of the angle $A_{n-1}A_nA_{n+1}$ is the intersection of the open half-plane bounded by the line $A_{n-1}A_n$ containing the point A_{n+1}, with the open half-plane bounded by the line A_nA_{n+1} containing the point A_{n-1}).