SPECIAL K
Saturday November 20, 1999
9:00 am - 12:00 noon

1: Given two circles C_1 and C_2 in the plane, find the locus of all points P for which the tangents from P to each of C_1 and C_2 have equal lengths.

2: How many sets of four distinct points forming the vertices of a trapezoid are there if the points are chosen from the vertices of a regular n-gon, where n is an integer ≥ 4?

3: Let a_i and c_i be positive numbers for $i = 1, 2, \cdots, n$. Prove that

$$\sqrt[n]{(a_1 + c_1)(a_2 + c_2)\cdots(a_n + c_n)} \geq \sqrt[n]{a_1 a_2 \cdots a_n} + \sqrt[n]{c_1 c_2 \cdots c_n}\,.$$

State when the equality is obtained.

4: Find all the integers that can be written in the form

$$\frac{1}{n_1} + \frac{2}{n_2} + \frac{3}{n_3} + \cdots + \frac{1999}{n_{1999}}$$

where $n_1, n_2, \cdots, n_{1999}$ are positive integers.

5: Show that for all positive integers n there exists a positive integer d such that

$$d, 2d, 3d, \cdots, nd$$

are all perfect powers. (A positive integer m is a perfect power if it can be written in the form j^i where j and i are positive integers with $i \geq 2$.)

BIG E
Saturday November 20, 1999
9:00 am - 12:00 noon

1: How many sets of four distinct points forming the vertices of a trapezoid are there if the points are chosen from the vertices of a regular n-gon, where n is an integer ≥ 4?

2: Prove or disprove: Suppose $P(x)$ and $Q(x)$ are two polynomials in a real variable x with $|P(x)|^2 - |Q(x)|^3 = 1$. Then P and Q must be constant polynomials (i.e. of degree zero).

3: Prove or disprove: It is possible to write every real-valued function $f(x, y, z)$ of three real variables as

$$f(x, y, z) = \psi\big(\phi(x, y), z\big)$$

where ψ and ϕ are appropriately chosen real-valued functions of two real variables.

4: A function $f : \mathbf{R} \to \mathbf{R}$ is said to be *convergence preserving* (CP) if for every convergent series $\sum a_n$, the series $\sum f(a_n)$ also converges.

Prove or disprove: If f is CP, then there exists a real number M and an $\epsilon > 0$ such that

$$\frac{f(x)}{x} < M$$

for all $0 < x < \epsilon$.

5: Show that for all positive integers n there exists a positive integer d such that

$$d, 2d, 3d, \cdots, nd$$

are all perfect powers. (A positive integer m is a perfect power if it can be written in the form j^i where j and i are positive integers with $i \geq 2$.)