**SPECIAL K**
**Saturday November 1, 1997**
**9:00 am - 12:00 noon**

**1:** Which positive integers are of the form

$$\frac{(x+y)^2}{xy}$$

for some positive integers $x$ and $y$?

**2:** A positive integer $n$ is called *logarithmically perfect* when

$$2\log n = \sum_{d|n} \log d\,.$$

Find all logarithmically perfect numbers between 1 and 100, inclusive.

**3:** Prove that $a^2 + b^2 + c^2 + ab + bc + ca$ is not a factor of

$$a^n(b-c) + b^n(c-a) + c^n(a-b)$$

for any integer $n \geq 5$.

**4:** Evaluate

$$\prod_{n=1}^{1997} \frac{n^2+1}{\sqrt{n^4+4}}\,.$$

**5:** Let $P$ be any point on the median from vertex $A$ to side $BC$ of the triangle $ABC$. Extend the line segment $BP$ to meet $AC$ at $D$. Similarly, extend the line segment $CP$ to meet $AB$ at $E$. If the circles inscribed in triangles $BPE$ and $CPD$ have the same radius, prove that $AB = AC$.

**BIG E**
**Saturday November 1, 1997**
**9:00 am - 12:00 noon**

**1:** Evaluate

$$\sum_{n=1}^{1997} \lfloor \log_2 n \rfloor ,$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to $x$.

**2:** For what real values of $a$ is

$$\frac{e^x + e^{-x}}{2} \leq e^{a\,x^2}$$

true for all real $x$?

**3:** Find all continuous functions $f : \mathbf{R} \to \mathbf{R}$ such that $f(x) = f(\cos x)$ for all real $x$.

**4:** For positive integers $m$ and $n$, prove that

$$\sum_{k=1}^{\infty} k^n \left( \tfrac{m}{m+1} \right)^k$$

is a positive integer.

**5:** Define

$$A_{n+1} = 1 + \frac{n}{A_n}$$

for integers $n \geq 1$, with $A_1 = 1$. Prove that

$$\tfrac{1}{2} + \sqrt{n - \tfrac{3}{4}} \ \leq A_n \ \leq \tfrac{1}{2} + \sqrt{n + \tfrac{1}{4}}$$

for all $n \geq 1$.