SPECIAL K
Saturday November 4, 1995
9:00 am - 12:00 pm

1: How many digits are there in the integer 2^{100} when it is multiplied out?

2: Find all real numbers $x > -1$ such that

$$(1+x)^x + (2+x)^x = (3+x)^x\,.$$

3: Prove that among any 13 distinct real numbers, it is possible to choose two, x and y, such that

$$0 < \frac{x-y}{1+xy} < 2-\sqrt{3}\,.$$

4: Let M be an $n \times n$ matrix whose entries are independent integers randomly chosen with replacement from the set $\{1, 2, 3, \cdots, 2^{1995}\}$. Find the probability that the determinant of M is an odd number.

5: Find all points (x, y) in the plane which lie on the union of the two hyperbolas

$$y^2 - xy - x^2 = \pm 1$$

where x and y are positive integers.

BIG E
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1: Evaluate

$$\prod_{n=1}^{\infty} \frac{e^{1/n}}{1+(1/n)}$$

in terms of well-known mathematical constants.

2: Find all real numbers $x > -1$ such that

$$(1+x)^x + (2+x)^x = (3+x)^x \, .$$

3: Let M be an $n \times n$ matrix whose entries are independent integers randomly chosen with replacement from the set $\{1, 2, 3, \cdots, 2^{1995}\}$. Find the probability that the determinant of M is an odd number.

4: Find all points (x, y) in the plane which lie on the union of the two hyperbolas

$$y^2 - xy - x^2 = \pm 1$$

where x and y are positive integers.

5: Suppose that $4^n + 2^n + 1$ is a prime number, where n is a positive integer. Prove that n must be a power of 3.