Bernoulli Trials Problems for 2018

1: There exists a positive integer k such that 2^k ends with the digits 2018 in its decimal representation.

2: There exists a positive integer n which is a multiple of 2018 such that the sum of the digits of n is equal to 2018.

3: There exist infinitely many positive integers n such that $(2018n)!$ is a multiple of $n!+1$.

4: When $n = 2018$, there exists a permutation σ of the set $\{1, 2, \cdots, 3n-1, 3n\}$ with the property that $\sigma(3k) = \sigma(3k-1) + \sigma(3k-2)$ for all $k \in \{1, 2, \cdots, n\}$.

5: For all positive integers a and b with $\gcd(a, b) = 1$, there exist infinitely many positive integers k such that $a + kb$ is a Fibonacci number.

6: For a polynomial of the form $f(x) = \sum\limits_{k=0}^{20} c_k x^k$ with each $c_k \in \mathbf{Z}$ and $c_0 = 20$ and $c_{20} = 3$, the largest possible number of distinct rational roots of $f(x)$ is equal to 6.

7: There exists a bijective function from the Euclidean plane to the open unit disc which sends lines in the plane to chords in the disc.

8: For every bounded function $f : \mathbf{R} \to \mathbf{R}$, if $f(x) + f\big(x + \frac{5}{6}\big) = f\big(x + \frac{1}{3}\big) + f\big(x + \frac{1}{2}\big)$ for all $x \in \mathbf{R}$ then f is periodic.

9: For all functions $u, v : \mathbf{R} \to \mathbf{R}$, if the function $f(x) = u\big(v(x)\big)$ is continuous then the function $u\big(-v(x)\big)$ is continuous.

10: There exists a bounded $\mathcal{C}^\infty$ function $f : \mathbf{R} \to \mathbf{R}$ such that $\lim\limits_{n\to\infty} f^{(n)}(0) = \infty$.

11: For every increasing function $f : (0, 1) \to (0, 1)$ with $f(x) > x$ for all $x \in (0, 1)$, there exists a continuous function $g : (0, 1) \to (0, 1)$ which is not increasing and has the property that $g(x) < g\big(f(x)\big)$ for all $x \in (0, 1)$.

12: There exists a 4×4 real-valued matrix A such that $A^4 = \begin{pmatrix} -1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & -2 \end{pmatrix}$.

13: There exists a 2×2 integer-valued matrix A such that the entries of A^2 are prime numbers and the determinant of A is the square of a prime number.

14: There exists a decreasing sequence of positive real numbers $\{a_n\}$ such that $\sum\limits_{n=1}^{\infty} a_n$ diverges and $\sum\limits_{n=1}^{\infty} n! a_{n!}$ converges.

15: There exists a sequence of complex numbers $\{a_n\}$ with the property that for all positive integers p, the sum $\sum\limits_{n=1}^{\infty} {a_n}^p$ converges if and only if p is a prime number.