Bernoulli Trials Problems for 2013

1: $1+1+1-1+1+1-1+1-1-1+1+1+1-1+1+1-1+1-1-1+1-1-1+1-1-1-1+1+1-1-1+1-1-1+1+1+1-1+1-2+1-1+1-1-1=0\,.$

2: There exist $a, b \in \mathbf{Q}$ such hat $a^b \in \mathbf{Q}$ but $b^a \notin \mathbf{Q}$.

3: It is possible to partition $\mathbf{Q}^+$ into two non-empty disjoint sets which are each closed under addition.

4: For all positive integers a, b and n, if $a|n$ and $b|n$ and $ab < n$ then $\gcd\left(\frac{n}{a}, \frac{n}{b}\right) > 1$.

5: For all continuous functions $f : \mathbf{R} \to \mathbf{R}$, if for every $0 \neq c \in \mathbf{R}$ the graph of $y = cf(x)$ is congruent to the graph of $y = f(x)$, then $f(x) = ax + b$ for some $a, b \in \mathbf{R}$. (Two sets are *congruent* when they are related by an isometry, that is a composite of translations, rotations and reflections).

6: For $n \in \mathbf{Z}^+$, let a_n be the number of congruence classes of triangles with integer sides and perimeter n. Then for every odd integer $n \in \mathbf{Z}^+$ we have $a_n = a_{n+3}$.

7: There exists a continuous map $f : [0,1] \to [0,\pi]$ such that f restricts to a bijection $f : \mathbf{Q} \cap [0,1) \to \mathbf{Q} \cap [0,\pi)$.

8: There exists a twice-differentiable function $f : \mathbf{R} \to \mathbf{R}$ with $f''(0) \neq 0$ with the property that $f'(x) = f(x+1) - f(x)$ for all $x \in \mathbf{R}$.

9: For all $n \in \mathbf{Z}^+$ and for all $n \times n$ matrices A and B, we have $e^{A+B} = e^A e^B$. (For an $n \times n$ matrix X, $e^X = I + X + \frac{1}{2!}X^2 + \frac{1}{3!}X^3 + \cdots$).

10: There exists a 3×3 matrix A over $\mathbf{Z}_2$ with $A \neq I$ and $A^7 = I$.

11: The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{1+1/\sqrt{\ln n}}}$ converges.

12: The series $\displaystyle\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} \frac{1}{k^2 + l^2}$ converges.

13: The sequence $\displaystyle\frac{1}{(\ln n)^2} \sum_{k=1}^{n} \left(\sqrt[k]{k} - 1\right)$ converges as $n \to \infty$.

14: The series $\displaystyle\sum_{n=2}^{\infty}\sum_{k=1}^{n-1} \left(\frac{k}{n}\right)^{kn}$ converges.

15: Euclidean space can be partitioned into a disjoint union of pairwise skew lines.