# Bernoulli Trials Problems for 2010

**1:** There exists a rational number $x$ with $x \neq \pm 1$ such that $x + \frac{1}{x}$ is an integer.

**2:** For every positive integer $n$ there exist positive integers $x$, $y$ and $z$ such that $x^2 + y^2 = z^n$.

**3:** If there exists a triangle with sides of lengths $a$, $b$ and $c$, then there also exists a triangle with sides of lengths $\sqrt{a}$, $\sqrt{b}$ and $\sqrt{c}$.

**4:** Let $p_n$ be the probability that a number selected at random from the set $\{1, 2, 3, \cdots, n\}$ has its leading digit equal to 1. Then $\lim\limits_{n\to\infty} p_n = \frac{1}{9}$.

**5:** If the sequence $\{a_n\}$ is bounded with $\lim\limits_{n\to\infty}(a_n - a_{n+1}) = 0$ then it converges.

**6:** There exists a bounded sequence $\{a_n\}_{n\geq 1}$ of real numbers with the property that for all $k > l \geq 1$ we have $|a_k - a_l| \geq \frac{1}{k-l}$.

**7:** There exists a quadratic $f(x) = ax^2 + bx + c$ with integral coefficients whose discriminant is equal to 23.

**8:** There exists a cubic $f(x) = ax^3 + bx^2 + cx + d$ with integral coefficients such that $f(19) = 1$ and $f(62) = 2$.

**9:** Let $f(x)$ be positive and continuous for $x \in [0, \infty)$. If $\displaystyle\int_0^\infty f(x)\,dx$ converges then so does $\displaystyle\int_0^\infty f(x)^2\,dx$.

**10:** There exist 100 positive integers whose sum is equal to their least common multiple.

**11:** There exist 7 distinct primes, all less than 900, which are in arithmetic progression.

**12:** Every open set in the plane is equal to the union of a set of disjoint non-degenerate closed line segments.

**13:** A rectangular box with sides of length 1, 2 and 3 hovers above the flat ground. The maximum possible area of its shadow on the ground is equal to 7.