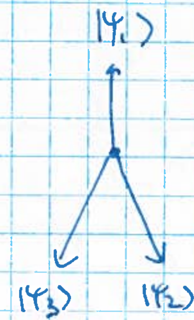
eg Let $|\psi_1\rangle = |0\rangle$

$|\psi_2\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$

$|\psi_3\rangle = \cos\theta\,|0\rangle - \sin\theta\,|1\rangle$



s.t.

$0.4\,|\psi_1\rangle\langle\psi_1| + 0.3\,|\psi_2\rangle\langle\psi_2| + 0.3\,|\psi_3\rangle\langle\psi_3| = \frac{I}{2}$

$|\psi_2\rangle, |\psi_3\rangle$ are closer to one another than to $|\psi_1\rangle$

Let $\rho_x = 0.9\,|\psi_x\rangle\langle\psi_x| + 0.1\,\frac{I}{2}$.

Consider Q-box emitting ρ_x when input $= x$, $x = 1, 2, 3$.

Then $C(Q) \doteq \max\limits_{p_1, p_2, p_3} S\left(\sum_{x=1}^{3} p_x \rho_x\right) - \sum_x p_x S(\rho_x)$.

Claim: optimal distribution is $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.3$.

Pf: Here $S(\rho_x)$ independent for x so we can focus on maximizing $S\left(\sum_x p_x \rho_x\right)$, which is $S\left(\frac{I}{2}\right) = 1$ attained for $p_1 = 0.4$, $p_2 = p_3 = 0.3$.

NB Each ρ_x has eigenvalues $0.95, 0.05$, so $S(\rho_x) \doteq 0.28640$

$\therefore C(Q) = 1 - 0.28640 \approx 0.71360$

What is the code?

Consider all strings of $1, 2, 3$ of length n with $1, 2, 3$ occurring $\approx 40\%, 30\%, 30\%$ of the time.

$H(\{0.4, 0.3, 0.3\}) \approx 1.571$ so there are $\approx 2^{1.571n}$ such strings

Strings are drawn with replacement, "almost" equiprobably, $\approx 2^{(0.71-\delta)n}$ times, from this set of $2^{1.571n}$.

For $n = 100$, $\dfrac{2^{0.71n}}{2^{1.57n}} \approx 2^{-0.86n} \approx 2^{-86}$ so a very sparse set has been chosen to be codewords even for small n.

Say, $C_1 = 211232 \cdots 1233$ $\longleftarrow$ highly correlated

$C_2 = 312211 \cdots 3123$

$\vdots$

$C_{2^{n(0.71-\delta)}} = 322111 \cdots 3211$

So Bob has to distinguish

$$\gamma_1 = \rho_2 \otimes \rho_1 \otimes \rho_1 \otimes \rho_2 \otimes \rho_3 \otimes \rho_2 \cdots \otimes \rho_1 \otimes \rho_2 \otimes \rho_3 \otimes \rho_3$$

$$\gamma_2 = \rho_3 \otimes \rho_1 \otimes \rho_2 \otimes \rho_2 \otimes \rho_1 \otimes \rho_1 \cdots \otimes \rho_3 \otimes \rho_1 \otimes \rho_2 \otimes \rho_3 .$$

$\vdots$

$$\gamma_{\underset{k}{2}^{n(0.71-\delta)}} = \rho_3 \otimes \overbrace{\rho_2 \otimes \rho_2 \otimes \rho_1 \otimes \rho_1 \otimes \rho_1 \cdots \otimes \rho_3 \otimes \rho_2}^{\pi_2} \otimes \rho_1 \otimes \rho_1$$

π_1 on $\rho_1^{\otimes n(0.44\ldots)}$

$$\Pi_{C_{2^{n(0.71-\delta)}}} = \underbrace{\pi_1 \otimes \pi_2 \otimes \pi_3}_{\text{on the appropriate sys. defined on } C_{2^{n(0.71-\delta)}}}$$

$$\Gamma_{C_{2^{n(0.71-\delta)}}} = \Pi \, \Pi_{C_{2^{n(0.71-\delta)}}} \, \Pi$$

$$\Gamma = \sum_{i=1}^{2^{n(0.71-\delta)}} \Gamma_{C_i}$$

$M_i = \Gamma^{-\frac{1}{2}} \Gamma_{C_i} \Gamma^{-\frac{1}{2}}$ $\leftarrow$ note heavily entangling

Finally $C(Q) \approx 0.71 \gg I_{acc}(\{p_x, \rho_x\})$ (cf 0.585 for the trines)