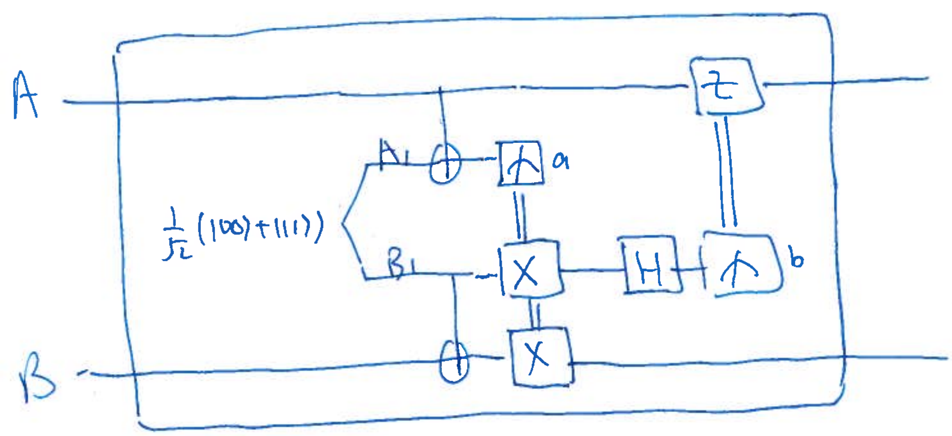
Proof sketch:

① Verify $(H\otimes I)\, CNOT_{AB}\, (Y^a \otimes Z^b)\, |\Phi\rangle = |b\rangle|a\rangle$

Then show that

$$CNOT_{AB} + 1\text{ ebit} \geq 1\text{ cobit}(\to) + 1\text{ cobit}(\leftarrow)$$

② Verify the following circuit by Gottesman that it performs a $CNOT_{AB}$.



$(X = \sigma_1,\ Z = \sigma_3)$

Show that if a, b are transmitted without being measured through 1 cobit(→) & 1 cobit(←) then we have:

$$1\text{ ebit} + 1\text{ cobit}(\to) + 1\text{ cobit}(\leftarrow) \geq 1\, CNOT_{AB} + 2\text{ ebits}.$$

Together, if 1 ebit can be borrowed and returned:

$$1\text{ cobit}(\to) + 1\text{ cobit}(\leftarrow) = 1\, CNOT_{AB} + 1\text{ ebit}.$$

③ Take 1 SWAP = 1 qbit$(\rightarrow)$ + 1 qbit $(\leftarrow)$

Then:

$$1\,\text{SWAP} + 2\,\text{ebits} = \begin{bmatrix} 1\ \text{qbit}(\rightarrow) \\ + \\ 1\ \text{ebit} \end{bmatrix} + \begin{bmatrix} 1\ \text{qbit}(\leftarrow) \\ + \\ 1\ \text{ebit} \end{bmatrix}$$

$$= 2\,\text{cobits}_{(\rightarrow)} + 2\,\text{cobits}_{(\leftarrow)}$$

$$= 2\,\text{CNOT}_{AB} + 2\,\text{ebits}.$$

$\therefore$ 1 SWAP = 2 CNOT_{AB} .