Fall 2012 QIC 890 / CO 781 Assignment 6

Due Nov 08, 2012 (in class)

Question 1. The quantum capacity of the amplitude damping channel [19 +1 marks]

Recall that the amplitude damping channel $\mathcal{N}_\gamma$ has a Kraus representation

$$\mathcal{N}(\rho) = A_0\rho A_0^\dagger + A_1\rho A_1^\dagger \text{ where } A_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} \text{ and } A_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}.$$

(a) [3 marks] Find an isometric extension U_γ for $\mathcal{N}_\gamma$. You can give a circuit, or specify the image of an arbitrary input. In the latter, justify why the mapping is isometric. In either case, explain very briefly why the mapping extends $\mathcal{N}_\gamma$.

(b) [2 marks] What is $\mathcal{N}_\gamma^c$?

(c) [4 marks] What is $\mathcal{N}_\eta \circ \mathcal{N}_\gamma$?

(d) [3 marks] Show that $\mathcal{N}_\gamma$ is degradable for a certain range of γ. Show what is this range, (and give a degrading map and show that it "degrades").

(e) [4 marks] Find the optimal state and $Q^{(1)}(\mathcal{N}_\gamma)$. Consider cases $\gamma \geq 1/2$ and $\gamma \leq 1/2$ separately.

You can assume without proof that, if $\mathcal{N}$ is degradable, $|\phi_i\rangle$ purifies ρ_i for $i = 0, 1$, then,

$$pI_c(R\rangle B)_{I\otimes\mathcal{N}(|\phi_0\rangle\langle\phi_0|)} + (1-p)I_c(R\rangle B)_{I\otimes\mathcal{N}(|\phi_1\rangle\langle\phi_1|)} \leq I_c(R\rangle B)_{I\otimes\mathcal{N}(|\phi\rangle\langle\phi|)}$$

where $|\phi\rangle$ purifies $p\rho_0 + (1-p)\rho_1$. Use this fact to show the optimal state has the form $(\sqrt{1-\beta}|00\rangle + \sqrt{\beta}|11\rangle)_{RA}$. You can leave the answer as an optimization over β for a simple enough function, and plot the optimal β as a function of γ.

(f) [1 mark] Find $Q(\mathcal{N}_\gamma)$.

(g) [3 marks] Using the proof of the direct coding theorem given in class, and following your answer in (d), describe what is Π_t.

Clearly, you don't need to describe the special properties that I didn't cover in class. I only informally discussed the method of types in class. The definitions are given in a couple of pages in the scanned notes.