Fall 2012 QIC 890 / CO 781 Assignment 1

Due Sept 20, 2012 (in class)

Question 1. Remote state preparation [10 marks]

Alice and Bob share 1 ebit in systems A and B.

Later on, Alice decides (in her mind) a pure 1-qubit state $|\psi_0\rangle = a|0\rangle + b|1\rangle$, where $a, b \in \mathbb{C}$ and $|a|^2 + |b|^2 = 1$. Then, she performs a measurement on system A along the basis:

$$|\psi_0\rangle = a|0\rangle + b|1\rangle, \quad |\psi_1\rangle = -b^*|0\rangle + a^*|1\rangle$$

where * represents the complex conjugate.

(a) Derive the probability for each measurement outcome and the corresponding postmeasurement state on system B.

(b) Suppose Alice only chooses $a, b \in \mathbb{R}$. Using your answer in (a) to propose a method to transmit $|\psi_0\rangle$ to Bob using 1 ebit and 1 cbit. Is the communication cost optimal? Why?

(c) Why doesn't the protocol in (b) contradict the optimality of teleportation? Give 2 reasons.

(d) What happens if Alice and Bob are allowed to use 1 cobit instead of 1 cbit?

Question 2. Impossibility to send quantum states via a classical channel [4 marks]

Prove that, $\forall n \in \mathbb{Z}^+$, n cbits $\not\geq$ 1 qbit.

Hint: show that if Alice can encode an arbitrary one-qubit state $|\psi\rangle$ into n qubits, send each via a classical channel to Bob, who can decode the outputs and retrieve $|\psi\rangle$, then, there is a method to clone an arbitrary qubit state.

Question 3. Communication using the CNOT [6 marks]

The gate $\text{CNOT}_{A\to B}$ acts on computation basis states as:

$$|00\rangle_{AB} \to |00\rangle_{AB}, \qquad |01\rangle_{AB} \to |01\rangle_{AB}$$
$$|10\rangle_{AB} \to |11\rangle_{AB}, \qquad |11\rangle_{AB} \to |10\rangle_{AB}$$

while the Hadamard gate H acts as:

$$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

Note that $(\text{H} \otimes \text{H})\ \text{CNOT}_{A\to B}\ (\text{H} \otimes \text{H}) = \text{CNOT}_{B\to A}$, so the two gates are interconvertible using local unitaries, thus have the same capacity for any nonlocal task.

(a) Show that $\text{CNOT}_{A\to B} \geq 1$ cobit$_\rightarrow$. (Thus $\text{CNOT}_{A\to B} \geq 1$ cobit$_\leftarrow$.)

(b) Show that $\text{CNOT}_{A\to B} + 1$ cbit ≥ 1 qbit. (Hint at the back.)

(c) Show that $\text{CNOT}_{A\to B} + 1$ cobit ≥ 1 qbit + 1 ebit.

Consider the initial state $(a|0\rangle + b|1\rangle)_A|0\rangle_B$. What happens after $\textsc{cnot}_{A\to B}$ followed by $\textsc{h}_A$ followed by computation basis measurement on A?