QIC890/CS867/CO781 Assignment 2

Due Monday January 31, 2020, 5:00pm

Instruction: Please submit to Crowdmark, placing the answer to each question in the right place.

Question 0. Important stabilizer codes [0 marks]

For the 4-, 5-, and 7-qubit codes covered in class, please work through the verification for the error correction properties, possible encoded Pauli operators, and generation of codewords. Do not submit solutions.

Question 1. Stabilizer code correcting X and Z errors [10 marks]

Consider a stabilizer S with the following generators:

$$\begin{array}{ccccccccc} G_1 & = & X & X & X & X & X & X & X \\ G_2 & = & I & I & I & Y & Y & Y & Y \\ G_3 & = & I & Y & Y & I & I & Y & Y \\ G_4 & = & Y & I & Y & I & Y & I & Y \end{array}$$

and the stabilizer code $T(S)$ associated with S.

(a) [2 marks] State the block length n, the number of encoded qubits k, and the distance d for the code. Provide a brief justification for the distance.

(b) [4 marks] Explain why this code corrects up to one X or Z error. What happens if one Y error occurs?

(c) [4 marks] Provide a set of valid encoded Pauli operators $\bar{X}_i, \bar{Z}_i$ for $i = 1, 2, \cdots, k$.

Question 2. Encoded Pauli generators and codewords [8 marks]

Let $n \geq 4$ be an even integer. Consider a stabilizer S with two generators, $X^{\otimes n}$ and $Z^{\otimes n}$.

(a) [2 marks] What is the block length, the number k of logical qubits, and the distance of the stabilizer code $T(S)$? Provide a brief justification for the distance.

(b) [2 marks] State the conditions for a valid set of operators to be the generators for the logical Pauli group $\bar{X}_1, \cdots, \bar{X}_k, \bar{Z}_1, \cdots, \bar{Z}_k$.

(c) [2 marks] Provide one such set of logical Pauli group generators so that each $\bar{X}_i$ is a tensor product of I's and X's of weight 2, and each $\bar{Z}_i$ is a tensor product of I's and Z's of weight 2.

(d) [2 marks] Use the answer in (c) to write down the codeword $|\bar{b}_1 \cdots \bar{b}_{n-2}\rangle$ where each $b_i \in \{0, 1\}$.

Hint: the answers to (c) and (d) should be very simple. For example, answer to (d) is a superposition of 2 computational basis states.