QIC890/CS867/CO781-486 Assignment 1

Due Friday February 02, 2024, 7:00pm

Instruction: Please submit to Crowdmark, placing the answer to each question in the right place.

Question 1. Bosonic code for amplitude damping [8 marks]

Consider an infinite-dimensional Hilbert space with a basis $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle, \cdots\}$ where $|j\rangle$ denotes a state with j excitations (e.g., j photons). Consider the amplitude damping channel $\mathcal{A}_\gamma(\rho) = \sum_k A_k \rho A_k^\dagger$ where

$$A_k = \sum_{j \geq k} \sqrt{\binom{j}{k}} \sqrt{(1-\gamma)^{j-k}\,\gamma^k}\; |j-k\rangle\langle j|$$

represents the loss of k excitations from the system. In particular,

$$A_0 = \sum_j (1-\gamma)^{\frac{j}{2}} |j\rangle\langle j|, \qquad A_1 = \sum_{j\geq 1} \sqrt{j\,(1-\gamma)^{j-1}\,\gamma}\; |j-1\rangle\langle j|\,.$$

(a) [4 marks] Show that the codespace with basis

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|40\rangle + |04\rangle)\,, \qquad |\psi_1\rangle = |22\rangle$$

is a QECC for the error set $\mathcal{E} = \{A_0 \otimes A_0, A_0 \otimes A_1, A_1 \otimes A_0\}$.

(b) [4 marks] Describe a valid decoding operation for this QECC.

Question 2. Approximate error correction [6 marks]

Consider the QECC $\mathcal{C}'$ with basis

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|4\rangle + |0\rangle)\,, \qquad |\psi_1\rangle = |2\rangle$$

and the error set $\mathcal{E}' = \{A_0, A_1\}$. Note that $\mathcal{C}'$ is *not* a QECC for $\mathcal{E}'$; provide a decoding operation $\mathcal{D}$ so that $\| \mathcal{D}(\mathcal{A}_\gamma(\rho)) - \rho \|_1 \approx O(\gamma^2)$.

Question 3. Stabilizer code correcting X and Z errors [10 marks]

Consider a stabilizer S with the following generators:

$$\begin{array}{ccccccccc} G_1 & = & X & X & X & X & X & X & X \\ G_2 & = & I & I & I & Y & Y & Y & Y \\ G_3 & = & I & Y & Y & I & I & Y & Y \\ G_4 & = & Y & I & Y & I & Y & I & Y \end{array}$$

and the stabilizer code $T(S)$ associated with S.

(a) [2 marks] State the block length n, the number of encoded qubits k, and the distance d for the code. Provide a brief justification for the distance.

(b) [4 marks] Explain why this code corrects up to one X or Z error. What happens if one Y error occurs?

(c) [4 marks] Provide a set of valid encoded Pauli operators $\bar{X}_i, \bar{Z}_i$ for $i = 1, 2, \cdots, k$.

Question 4. Encoded Pauli generators and codewords [6 marks]

Let $n \geq 4$ be an even integer. Consider a stabilizer S with two generators, $X^{\otimes n}$ and $Z^{\otimes n}$.

(a) [1 mark] What is the block length, the number k of logical qubits, and the distance of the stabilizer code $T(S)$? Provide a brief justification for the distance.

(b) [3 marks] Provide one such set of logical Pauli group generators so that each $\bar{X}_i$ is a tensor product of I's and X's of weight 2, and each $\bar{Z}_i$ is a tensor product of I's and Z's of weight 2.

(c) [2 marks] Use the answer in (c) to write down the codeword $|\bar{b}_1 \cdots \bar{b}_{n-2}\rangle$ where each $b_i \in \{0, 1\}$.

Hint: the answers to (c) and (d) should be very simple. For example, answer to (d) is a superposition of 2 computational basis states.