Problem Set #5
Quantum Error Correction
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Problem #1. Correctness of Bell Measurement Extended Rectangle
For this problem, imagine you have a gadget for the two-qubit Bell measurement.

a) Define what it means for Bell measurement gadget to be fault-tolerant.

b) Define correctness for a Bell measurement extended rectangle.

c) Prove that a good Bell measurement ExRec is correct.

Problem #2. Pseudo-thresholds
For this problem, use the FTEC circuit for the 7-qubit code for which we computed the threshold in class, which contains 96 CNOT locations and 170 single-qubit locations (waits, Hadamards, measurements, and $|0\rangle$ preparations). Assume all single-qubit locations have the same error rate $p_{\text{single}}$, but CNOT locations may have a different error rate $p_{\text{CNOT}}$.

a) Write down formulas for the logical error rate (i.e., after level reduction) for a single level of the 7-qubit code for logical CNOT and logical single-qubit locations in terms of $p_{\text{single}}$ and $p_{\text{CNOT}}$. Lump together different types of single-block extended rectangles; the largest single-block extended rectangle is for the Hadamard.

b) Now imagine that the physical error rate for single-qubit locations is $p_{\text{single}} = 0$. Calculate the pseudo-threshold for CNOT gates: $p_{\text{PT}}$ is the CNOT error rate at which the logical CNOT error rate after one level of the QECC is less than or equal to $p_{\text{CNOT}}$.

c) Show that after two levels of concatenation, the logical CNOT error rate is greater than $p_{\text{PT}}$ when the error rates on physical locations are $p_{\text{single}} = 0$, $p_{\text{CNOT}} = p_{\text{PT}}$.

Problem #3. Threshold estimate based on Shor EC

a) Consider the following specific implementation of the Shor EC gadget associated with the 7-qubit Steane code, and count the number of locations.

In this implementation, the syndrome measurement is repeated 3 times. If the first two syndromes agree, or if the last two syndromes agree, a correction is made based on this majority. Otherwise, a correction is made based on the last syndrome.

Each 4-qubit cat state is to be prepared based on the following circuit (you may need to turn the page) and you should arrange the operations to minimize the number of wait locations involved:

b) How does the number of locations above compare to that using the Steane EC (covered in the lecture)?

c) Use your answer in (a) to lower bound the threshold for the CNOT ExRec.

d) Uncredited part: convince yourself the implementation in (a) satisfies ECCP and ECRP.
Measurements are along $|0\rangle, |1\rangle$. $a = 1$ iff $b_1 = 1$ and $b_2 = 0$. 