Problem Set #4
Quantum Error Correction
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Due Tuesday, Feb. 14, 2012

Problem #1. Repeating Syndrome Measurements

In this problem, we will consider different rules for repeating the error syndrome measurement for Shor error correction, and determine how many times the syndrome needs to be repeated in various methods.

a) First consider the case of a code which only corrects one error \( t = 1 \). Suppose we repeat the syndrome measurement \( r \) times and take the result that appears most frequently. (If necessary, suggest a rule for how to decide when there is a tie for most frequent result.) What is the smallest \( r \) for which the error correction step satisfies FT properties ECCP and ECRP?

b) Again for the \( t = 1 \) case, consider a procedure where we repeat until the measurement returns the same syndrome \( r \) times in a row. Now what value of \( r \) do we need to take to satisfy FT properties ECCP and ECRP? Assuming there is only 1 error in the course of the full EC procedure, what is the maximum number of times we will need to measure the syndrome?

c) Answer the questions from parts a and b again, but now for a code with an arbitrary \( t \).

Problem #2. Small circuit for cat state preparation

Show that, in the following circuit to prepare the 5-cat state, if at most one fault happens, it is impossible to accept an output state that has an error of weight 2 or above.

```
\[ \begin{array}{c}
|0\rangle \\
|0\rangle \\
|1\rangle \\
|1\rangle \\
|0\rangle \\
\end{array} \quad \begin{array}{c}
\times \\
\times \\
\times \\
\times \\
\times \\
\end{array} \quad \begin{array}{c}
+1 \\
-1 \\
\times \\
\times \\
\times \\
\end{array} \quad \begin{array}{c}
|0\rangle \\
|1\rangle \\
\times \\
\times \\
\times \\
\end{array} \]
```