1.9, part II

Moving on to transmit quantum data via quantum channels.
As in the case to send classical data via a classical/quantum channel, noise degrades the transmitted state, and error-correcting codes are used to suppress the degradation (at some trade-off).

**Quantum capacity (1)**

- Given quantum channel $N$, we say $R$ is achievable rate.
- Quantum state $W$ with $m$ uses of $N$, over $E^n$.
- $E^n \rightarrow E^m$.
- Model: $\rho \rightarrow N(\rho)$.
- $\rho \rightarrow \rho_K$.
- $E^n$.
- Achievable rate $R$.

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**Problem:** how to define $\hat{R}$? Is it $\hat{R}$? There are many defns but lacking they grant the same capacity.
See eg. 03.1087 by kreutzer & terhal.

We can demand any of the following:

1. \( (\hat{R}) \rightarrow \left\langle \hat{R} \right\rangle \)
2. Average $\left\langle \hat{R} \right\rangle$ and $\left\langle \hat{R} \right\rangle$.
3. $\left\langle \hat{R} \right\rangle$, $\left\langle \hat{R} \right\rangle$.
4. For $\left\langle \hat{R} \right\rangle$, $\left\langle \hat{R} \right\rangle$.
5. $\left\langle \hat{R} \right\rangle$, $\left\langle \hat{R} \right\rangle$.

Can also replace $\left\langle \hat{R} \right\rangle$ by $\| \hat{R} \hat{R} - f \|_R$.

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**Def 1** resolves about "preserving" the initial state, traditionally the easier one to show / think about.

**Def 2** resolves about $\hat{R}$, "simulating" the identity channel on $\mathbb{C}^n$-dim Hilbert space.

When used with the trace distance criteria, 2b tells us how good the simulation is, measured by the trace distance of the "Choi-Jamshidi states" of the simulating & the simulated channels.

CJ state of $N$ is just $\rho_{CJ} = \frac{1}{2}(\mathbb{I} \otimes N)$, for $\mathbb{I}$ being the identity.

This completely specifies the channel $N$.

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1.8 tells use the distance between the simulated & the simulating channel in the "diamond norm".

$\| N_1 - N_2 \|_\diamond := \max \| I \otimes N_1 - I \otimes N_2 \|_{1}\delta$

2 channels having small diamond norm is the most demanding way to say they're similar.

Aside: some researchers use the "cb-norm" instead of the diamond norm.

Besides the surprising relation $\delta$, $N$ is also "summarized" in a "Choi-Jamshidi" state:

$N|\psi\rangle = \sum_k E_k |\psi\rangle$ where $\sum_k E_k = I$.

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$N^*|\psi\rangle = \sum_k A_k^* |\psi\rangle$ is called the "adjoint" of $N$.

In the classical case:

In the quantum case:

The diamond & cb norm are similar when measuring the accuracy of approximating.

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On: What is an $(M, n)$ code for quantum data?

*Better view:

Quantum case: & Error correction

Ex: You can have different errors $E_1, E_2$, taking $C$ to the same space.

Identifies "deceased" C is called "degenerate.

Ex: Subspace $C$ & $C'$. Connected to the subspace $C$.

"Lossy errors" lead to new codes.
The LSD then:

\[ Q(N) = \sup_n Q^n(N) \]

In other words:

\[
\left| \begin{array}{c}
B_1 \\
B_2 \\
B_3 \\
B_4 \\
B_5 \\
B_6 \\
B_7 \\
B_8 \\
B_9 \\
B_{10} \\
B_{11} \\
B_{12} \\
B_{13} \\
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B_{60} \\
\end{array} \right| \quad (19x20) \]

* Calculate \( S(b_n, b_{n+1}) - S(b_{n+1}, b_n) \)
* Divide by \( n \)
* Sup over \( n \).

When transmitting classical data, we have \( I(R; E) \) (entangled in \( N(N^e) \)).

For quantum data, we only need to make \( R \) correlation with \( E \), hence take out a portion of this data that's correlated with \( E \). The QMI counts all correlation (classical + quantum) and the classical portion is quantified by \( S(R; E) \).

\[ I_c(R; E|E) = \frac{1}{2} (3-1) = 1 \quad \text{(by entropy.)} \]

So, what is \( I_c(R; E|b_n) \)?

\[
\begin{array}{c}
\text{Input}
\end{array}
\begin{array}{c}
R
\end{array}
\begin{array}{c}
\text{Output}
\end{array}
\begin{array}{c}
E
\end{array}
\]

Pure state \( I\)-states.

\[
I_c(R; E) = S(E) - S(R|E) = S(E) - S(E|E)
= \frac{1}{2} (S(E) - S(R|E) + S(R|E) + S(E|E))
= \frac{1}{2} (S(E) - S(E|E))
\]

\[ S(E; B) = 3, \quad S(R; E|E) = 1 \]
Properties of the subset information $I(R;B)$:

1. Invariant under local wiring on $R$ and $B$
2. Invariant under adding ancillas in pure states
3. Non-increasing under local operation on $R$'s side

If an auxiliary spigot $x$ on $R$:

\[ S(R;B) = S(R';B') \]

\[ (S(R;B) - S(R';B')) = 0 \]

\[ I(R;B) = I(R';B') \]

Example 1:

Given initial state $|x\rangle|0\rangle$ for $R$:

\[ I(R;B) = 1 \]

Magnitude of processing capacity

Example 2:

Circuit $A$:

\[ I(R;B) = 1 \]

Circuit $B$:

\[ I(R;B) = 0 \]