Figure 6.5. QUANTIFYING UNCERTAINTY: Confidence Intervals

Program 19 in: *Against All Odds: Inside Statistics*

Programs 19 through 25 are devoted to formal statistical inference. Formal inference attaches a probability statement to its conclusions to indicate how ‘reliable’ they are. There are two important types of inference: *tests of significance* and *confidence intervals*. This program introduces the basic ideas of confidence intervals and their interpretation in a simple setting. Later programs give specific confidence intervals for use in more realistic situations.

A 95% confidence interval for a model parameter such as the mean μ, or a population average $\bar{\mathbf{Y}}$, is an interval, computed from sample *data*, that contains the unknown ‘true’ value of μ or $\bar{\mathbf{Y}}$ in 95% of all possible samples from the population. The confidence statement has *two* parts: the interval itself and the *level of confidence*. The level of confidence is usually chosen to be 90%, 95% or 99%. In formulae, we write the confidence level as $1-\alpha$, so that α is usually .10, .5 or .01. The confidence level states the probability that the method will give a ‘correct’ answer; *i.e.*, if you use 95% confidence intervals often, in the long run 95% of your intervals will contain the ‘true’ parameter or attribute value. However, you cannot know whether the result of applying a confidence interval to a *particular* set of data is correct. The video points out that confidence intervals lie behind the "margin of error" that news reports often attach to opinion poll results.

In this program, we consider the confidence intervals for the model mean μ representing the average of a population. We assume the population is normally distributed and we have selected by SRS (*i.e*., EPS) a sample of size n from the population; these assumptions are often met quite closely in practice. We also assume that we know the standard deviation σ of the population; this is *not* realistic, but we want to illustrate the ideas of inference in a simple case. Program 21 describes what to do when σ is not known. The video looks at testing batteries, such as those used in heart pacemakers, to estimate their mean lifetime quite closely; the standard deviation is of course not known, but the ideas of confidence intervals apply nonetheless.

$$\bar{y} \pm z^*\sigma\sqrt{\frac{1}{n}}$$

A level $1-\alpha$ confidence interval for the mean μ of a normal population with known standard deviation σ, based on a sample of size n selected by SRS, is given by the expression at the right; here z^* is the *upper normal critical value* for $p = \alpha/2$, given in the table at the lower right of the first side of Figure 5.4 of the Course Materials. For a 95% confidence interval, for example, we *ex*clude the extreme 5% (α is .05); half of this, or 2.5%, falls in *each* tail of the distribution, so we look up the point z^* with probability 0.025 lying above it. [See the centre diagram at the bottom of the first side (page 5.9) of Figure 5.3.]

The width of a confidence interval shows how ‘accurate’ our estimate is; *small* widths are desirable. Other things being equal, the width of a confidence interval decreases as:

- the confidence level $1-\alpha$ *de*creases;
- the standard deviation σ *de*creases;
- the sample size n *in*creases.

$$n = \left(\frac{2z^*\sigma}{w}\right)^2$$

If we have chosen a confidence level and know the population standard deviation σ, we can get a confidence interval of any width we want by picking the sample size n. The sample size required to obtain a confidence interval of specified width w for a normal mean is as given at the right, where z^* is the critical point for the desired level of confidence. In *practice*, extra observations can be expensive or impractical. The video visits a primate research centre, where monkeys and other primates for use in research are housed. The task of deciding how many primates to use in a medical study is difficult both for ethical reasons and because of the expense involved.

A particular form of confidence interval is correct only under specific conditions. The most important conditions concern *the method used to produce the data*. Other factors such as the form of the population distribution may also be important.

1. In the first, second and fifth paragraphs above, the words *reliable*, *true*, *correct* and *accurate* are given in quotes (‘ ’); explain briefly the reason(s) behind this notation.
   - Explain briefly why the word *correct* in the first sentence of the *last* paragraph above is *not* given in quotes.

2. You are a statistical consultant to a business organization and, during your first meeting with the organization’s contact person, she tells you she only wants confidence intervals at the 99% level of confidence because it is obviously better to be 99% sure than only 95% or 90%. What would you say in reply to this assertion?

3. Show how the *lower* expression (for n) given at the right above is derived from the *upper* expression for a confidence interval.

4. One characteristic of the method used to produce the data, mentioned in the second sentence of the last paragraph of the summary of the video contents given above, is that the data should come from a sample obtained by equiprobable (random) selecting. Explain briefly why equiprobability in the selecting process is important.

5. Explain briefly the implication(s) of the third sentence of the last paragraph in the summary given above .... *the form of the population distribution may also be important.*

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