Figure 5.6. PROBABILITY MODELLING: Normal Distribution Examples

The following problems illustrate the use of the normal distribution as a probability model. A selection of the problems will be discussed in class; *you* should try to solve the *others*, remembering that the clarity of the presentation of your solution may be as, or more, important than the final answer. Also remember to give any *final* numerical answer(s) to an appropriate number of significant figures in light of the approximate nature of *all* mathematical models of real phenomena.

Example 5.6.1: If the random variables $Y \sim N(6, 4)$, find: $\Pr(Y \geq 8)$; $\Pr(Y \leq 4)$; $\Pr(3 < Y \leq 7)$.

Example 5.6.2: If the random variable $V \sim N(10, \sqrt{2})$, find the value of the constant c such that $\Pr(V > \text{c}) = 0.02$.

Example 5.6.3: If W and Y are random variables with $W \sim N(3, 2)$ and $Y \sim N(6, 3)$:

(a) evaluate: $\Pr(W \leq 2)$; $\Pr(1 < W \leq 2)$; $\Pr(W > 0)$; $\Pr(|W - 3| > 2)$; $\Pr(|W - 2| > 2)$;

(b) find values for b, c, d such that: $\Pr(Y \leq \text{b}) = 0.05$; $\Pr(Y > \text{c}) = 0.4$; $\Pr(|Y - 6| \leq \text{d}) = 0.95$.

Example 5.6.4: The examination scores obtained by a large group of students can be modelled by a normal distribution with a mean of 65% and a standard deviation of 10%. Using this model, find the percentage of students who obtain each of the following letter grades:

A (≥80%); B (70-80%); C (60-70%); D (50-60%); F (<50%).

Example 5.6.5: The light bulbs used in a large building have lifetimes that can be modelled by a normal distribution with a mean of 500 hours and a standard deviation of 50 hours. To minimize the number of bulbs failing during working hours, *all* bulbs are replaced after t hours of operation. What value of t should be used so that no more than 1% of bulbs fail before replacement?

Example 5.6.6: The mesh size s of a gill net is the minimum diameter of the fish which can be caught in the net. Fish diameters can be modelled by a $N(6, 2)$ distribution for one-year-old fish, by a $N(8, 2)$ distribution for two-year-old fish, and by a $N(10, 2)$ distribution for three-year-old fish.

(a) What should be the value of s to catch 80% of three-year-old fish?

(b) What proportion of two-year-old fish will be caught with the mesh size found in (a)?
What proportion of one-year-old fish?

Example 5.6.7: A creamery produces a large number of packages of butter; it is known from experience that the weights in grams of the packages can be modelled by a normal distribution with a standard deviation of 5 grams. At what value should the *average* package weight be set so that at least 90% of the packages weigh at least 450 grams?

Example 5.6.8: Suppose that the diameters in millimetres of the eggs laid by a large flock of hens can be modelled by a normal distribution with a mean of 40 mm and a standard deviation of 2 mm. The selling price per dozen is \$1.84 for eggs less than 37 mm in diameter, \$2.16 for those with diameters greater than 42 mm, and \$2.02 for the remainder. What is the average selling price *per egg* produced by the flock?

Example 5.6.9: A diagnostic test for a certain disease is based on a continuous measurement represented by random variable Y. Y can be modelled by a $N(100, 4)$ distribution for people *with* the disease and by a $N(80, 3)$ distribution for people *without* it. An individual is classified as diseased if $Y > \text{c}$ and as healthy if $Y \leq \text{c}$; thus, a person is *misclassified* if the test indicates $Y \leq \text{c}$ for a diseased individual or $Y > \text{c}$ for someone without the disease. If 20% of those tested have the disease, express the probability of misclassification as a function of c. By plotting this function, or otherwise, find the value of c which minimizes the probability of misclassification.

HINT: Pr(misclassification) = Pr(misclassification when diseased) × Pr(diseased) +
Pr(misclassification when not diseased) × Pr(not diseased).

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(*continued overleaf*)

The *answers* to the problems overleaf are as follows:

Example 5.6.1: 0.3085 ; 0.3085 ; 0.3721.

Example 5.6.2: $c = 12.9044 \simeq 12.90$.

Example 5.6.3: (a) 0.3085 ; 0.1498 ; 0.9332 ; 0.3174 ; 0.3753;
(b) $b = 1.0653$; $c = 6.7599$; $d = 5.8800$.

Example 5.6.4: *A*, *F*: $6.68 \simeq 7\%$; *B*, *D*: $24.17 \simeq 24\%$; *C*: $38.30 \simeq 38\%$.

Example 5.6.5: $t = 383.685 \simeq 384$ hours.

Example 5.6.6: (a) $s = 8.3168 \simeq 8.32$;
(b) two-year-old: $43.707 \simeq 43.7\%$; one-year-old: $12.3348 \simeq 12.3\%$.

Example 5.6.7: $456.408 \simeq 456.4$ grams.

Example 5.6.8: $16.9182 \simeq 16.92$¢ per egg.

Example 5.6.9: We can tabulate the probability of misclassification for different values of c as follows:

c	Pr(misclass.)
87	0.008 035 4
88	0.003 304 4
89	0.001 676
90	0.001 589 3
91	0.002 537

c	Pr(misclass.)
89.4	0.001 496 386
89.5	0.001 483 356
89.55	0.001 481 306
89.565	0.001 481 204
89.6	0.001 481 8
89.65	0.001 485 706
89.7	0.001 491 994

Thus, $c \simeq 89.565$ approximately minimizes the probability of misclassification at around 0.001 4812