Figure 5.15. PROBABILITY MODELLING: Linear Combinations Examples

The following problems illustrate the use of linear combinations (in particular, sums, differences and averages) of probabilistically independent normal random variables as probability models. A selection of the problems will be discussed in class; *you* should try to solve the *others*, remembering that the clarity of the presentation of your solution may be as, or more, important than the final answer. Also remember to give any *final* numerical answer(s) to an appropriate number of significant figures in light of the approximate nature of *all* mathematical models of real phenomena.

Example 5.15.1: A passenger elevator has a load limit of 10 people or 1,750 pounds. If the weights of the people who use the elevator can be modelled by a normal distribution with a mean of 165 pounds and a standard deviation of 10 pounds, find the probability the load limit will be exceeded by a randomly-chosen group of 10 people.

Example 5.15.2: Copper pipes produced by a certain machine have *external* diameters that can be modelled by a normal distribution with mean $\mu_1 = 0.498$ inches and standard deviation $\sigma_1 = 0.005$ inches. Another machine produces the corresponding pipe sleeves or junctions, for which the *internal* diameters can be modelled by a normal distribution with mean μ_2 (for which the value can be set by the machine's operator) and standard deviation $\sigma_2 = 0.004$ inches.

(a) If $\mu_2 = 0.510$ inches, find the probability a randomly-selected pipe will fit inside a randomly-selected sleeve or junction with a total clearance of not more than 0.02 inches.

(b) To what value should μ_2 be set to *maximize* the probability a pipe and a sleeve or junction will fit together as specified in (a)?

(c) Comment briefly on your value of μ_2 in (b) in relation to a suitable combination of two other quantities given in the problem statement.

Example 5.15.3: In a factory which manufactures beam balances from components, each balance is assembled by attaching a randomly-chosen pan and pan-holder to each end of the balance arm. The distribution of pan weights in grams can be modelled by a $N(50, \sqrt{0.0005})$ distribution, and the model for the weights of the pan-holders is $N(10, \sqrt{0.0003})$. A balance is *un*acceptable if the combined weights of the pan and pan-holder on each side of the balance differ by more than 0.075 gm. What proportion of the balances manufactured in the factory will be unacceptable?

Example 5.15.4: Three parts – two *A*'s and one *B* placed end-to-end – are needed to make up a machine shaft assembly; the lengths of the parts can be modelled by normal distributions with respective means of 6.00 and 35.20 cm and standard deviations of $\sqrt{0.0003}$ and $\sqrt{0.0004}$ cm. The assembly must then be placed in a case for which the inside length can be modelled by a normal distribution with a mean of 47.35 cm and a standard deviation of $\sqrt{0.0006}$ cm.

(a) If specifications call for the shaft assembly to fit inside the case with a clearance of between 0.1 and 0.2 cm, find the proportion of the final products that have the proper fit.

(b) From your answer to (a), what do you conclude about component tolerances for assembling multi-component systems successfully? Explain briefly.

(c) Identify where an assumption of *independence* is involved in your calculations in (a) and discuss briefly in each case how well the assumption would be likely to be met in practice.

(d) Recalculate the probability of a correct fit [as in (a)] if the average for the inside length of the case is (i) 47.30 cm; (ii) 47.40 cm. Assuming this average length can be adjusted in the manufacturing process for the case, discuss briefly the matter of general interest these two probabilities illustrate when compared with that in (a).

Example 5.15.5: The random variables $Y \sim N(10, \sqrt{2.5})$ and $Z \sim N(12, \sigma)$ are probabilistically independent; if $\Pr(Y > Z) = 0.85$, find the value of σ.

Example 5.15.6: If the personal incomes of people living in a particular city can be modelled by a normal distribution with mean \$37,200 and standard deviation \$800, find the probability that, in a sample of 64 people selected equiprobably from the city, the average income exceeds \$37,400.

Example 5.15.7: The examination scores obtained by a large group of students can be modelled by a normal distribution with a mean of 65% and a standard deviation of 10%. Using this model:

(a) find the probability the average score of a randomly-chosen group of 25 students is greater than 70%;

(b) find the probability the average scores of two distinct randomly-selected groups of 25 students differ by more than 5 marks.

(*continued overleaf*)

Example 5.15.8: Suppose the pH of rain in Muskoka can be modelled by a normal distribution with mean 5.8 and standard deviation 0.3 units.

(a) Find the probability the pH of the *average* of four rainfalls selected equiprobably lies within 0.1 units of the (true) mean.

(b) How many rainfalls must be selected equiprobably to be 95% sure their average pH lies within 0.1 units of the mean.

Example 5.15.9: The random variable $\overline{Y}$, representing the average of the response variate of a sample of elements obtained by equiprobable selecting, is to be used as an estimate of the mean μ of a normal distribution whose standard deviation is known to be 8 cm. Find the size, n, of the sample so that, with a probability of 90%, the estimate ($\overline{y}$) differs from the true value (μ) by at most 1 cm.

Example 5.15.10: The measurement error of a process for measuring length can be modelled by a normal distribution with mean 0 and standard deviation σ. The process is to be used to measure the lengths of two (different) objects, a total of only *two* measurements being allowed.

(a) If each object is measured once, what is the *imprecision* of the process for measuring each length?

(b) Alternatively, if the two measurements are the *sum* and the *difference* of the two lengths, show that the imprecision of the process for measuring the individual lengths is *decreased*.

(c) Discuss briefly whether the procedure described in (b) represents a *practical* method for decreasing the imprecision of the measuring process for a quantity like length.

The *answers* to the problems overleaf and above are as follows:

Example 5.15.1: 0.000 7827 ≃ 0.08%.

Example 5.15.2: (a) 0.8637 ≃ 86.4%.

(b) $\mu_2 = 0.508$ inches (the corresponding probability is 0.8816).

(c)

Example 5.15.3: 0.06079 ≃ 6.1%.

Example 5.15.4: (a) 0.78870 ≃ 79%.

(b)

(c)

(d) 0.494 614 ≃ 49½%.

Example 5.15.5: $\sigma = 1.106\ 328 \simeq 1.106$.

Example 5.15.6: 0.0228 ≃ 2.3%.

Example 5.15.7: (a) 0.00621 ≃ 0.6%.

(b) 0.07710 ≃ 7.7%.

Example 5.15.8: (a) 0.49500 ≃ 50%.

(b) 34.574 ≃ 35 rainfalls.

Example 5.15.9: 173.165 ≃ 174 observations.

Example 5.15.10: (a) The imprecision is σ.

(b) The imprecision is now $\sigma\sqrt{\frac{1}{2}}$

(c)