Figure 5.13. PROBABILITY MODELLING: Exponential Distribution Examples

The following problems illustrate the use of the exponential distribution as a probability model. A selection of the problems will be discussed in class; *you* should try to solve the *others*, remembering that the clarity of the presentation of your solution may be as, or more, important than the final answer. Also remember to give any *final* numerical answer(s) to an appropriate number of significant figures in light of the approximate nature of *all* mathematical models of real phenomena.

Example 5.13.1: If the random variable $T \sim Exp(\theta = 2.5)$, find: $\Pr(-6 < T \le 4)$; $\Pr(T \le 4)$; $\Pr(T > 1)$.

Example 5.13.2: n electronic components in a satellite are wired in such a way that all start operating at the same time but all must fail for the satellite to fail. If each component has a time to failure that can be modelled by an exponential distribution with mean 20 years, how many of the components must be used so the satellite operates for d years with probability π?

Example 5.13.3: My neighbour and I have identical floodlamps with lifetimes that can be modelled by an exponential distribution with mean $\theta = 300$ hours. If we both burn our lamps for 6 hours each night, find the probability:

(a) my floodlamp lasts longer than 60 nights;
(b) both floodlamps last longer than 60 nights;
(c) both floodlamps burn out on the same night.

HINT: For (c), express the probability as an infinite sum, in which each term is obtained by reasoning as in (b); this sum can be evaluated as an infinite geometric series.

Example 5.13.4: The p.d.f. of the *double exponential distribution* is $f(y) = \mathrm{k}e^{-|y|}$; $-\infty < y < \infty$.
Evaluate the constant k and neatly sketch the p.d.f.

Example 5.13.5: The time from treatment to recurrence of a certain type of cancer can be modelled by an exponential distribution with mean θ. Fifty percent of patients are observed to have a recurrence of the cancer within 693 days of the completion of treatment. Estimate the parameter θ and find the probability a patient who has had no recurrence in the first year after treatment will have a recurrence during the following year.

HINT: The *conditional probability of A given B* is: $\Pr(A|B) = \Pr(A \cap B)/\Pr(B)$.

Example 5.13.6: For a certain type of electronic component, the lifetime, L, in thousands of hours can be modelled by an exponential distribution with mean $\theta = 2$.

(a) Find the probability a randomly-selected component will last longer than 1,000 hours.
(b) If a component has *already* lasted 1,000 hours, find the probability it will last at least another 1,000 hours.
(c) Find the expected *total* lifetime of a component which has already lasted for 1,000 hours.

Example 5.13.7: A manufacturer must choose between two processes for producing components. The random variable L, the length of a component in centimetres, has the p.d.f. shown at the right. Only components with lengths between 1.1 and 2.0 cm are acceptable.

Process 1: $f(l) = 3/l^4$; $1 < l < \infty$
Process 2: $f(l) = 4/l^5$; $1 < l < \infty$

(a) Which process produces the greater proportion of acceptable components?
(b) What is the average length of the components from Process 1?
(c) What is the average length of all *acceptable* components from Process 1?

(*continued overleaf*)

The *answers* to the problems overleaf are as follows:

Example 5.13.1: $1 - e^{-1.6} \simeq 0.7981$; $1 - e^{-1.6} \simeq 0.7981$; $e^{-0.4} \simeq 0.6703$.

Example 5.13.2: $n = \dfrac{ln(1-\pi)}{ln(1-e^{-d/20})}$;

substituting selected values of π and d into this expression, we obtain the following values for n:

π	d = 5	d = 10	d = 15	d = 20	d = 25
0.90	1.53	2.47	3.60	5.02	6.82
0.95	1.99	3.21	4.69	6.53	8.87
0.99	3.05	4.94	7.20	10.04	13.64
0.999	4.58	7.41	10.80	15.06	20.46.

Example 5.13.3: (a) $e^{-6/5} \simeq 0.3012$;

(b) $[e^{-6/5}]^2 \simeq 0.0907$ (assuming independence);

(c) $\dfrac{1-e^{-1/50}}{1+e^{-1/50}} \simeq 0.009\,999\,667 \simeq 0.010$.

Example 5.13.4: k = ½ so the p.d.f. is: $f(y) = \begin{cases} \tfrac{1}{2}e^{-y} & ; \quad y \geq 0 \\ \tfrac{1}{2}e^{y} & ; \quad y < 0 \end{cases}$

Example 5.13.5: $\theta = \frac{693}{\ln 2} \simeq 999.79 \simeq 1{,}000$ days;

Pr(recurrence in *2*nd year, given no recurrence in *1*st year) $= 1 - e^{-365/\theta} \simeq 0.3059$.

Example 5.13.6: (a) $e^{-1/2} \simeq 0.6065$;

(b) $e^{-1/2} \simeq 0.6065$;

(c) 3 (thousand hours).

Example 5.13.7: (a) Process 1 is *slightly* higher (0.626 315 ≃ 62.6% *vs.* 0.620 513 ≃ 62.1% acceptable);

(b) 1.5 cm;

(c) 1.3806 cm.