Assignment 4

A4 – 1. (a,b) Text Exercise 6.1 (page 447): *You measure the weights of 24 male runners. You do not actually .....*
(c) Text Exercise 6.4 (page 448): *Find a 99% confidence interval for the average weight* μ .....

A4 – 2. (a-d) Text Exercise 6.2 (page 447-448): *Crop researchers plant 15 plots with a new variety of corn. .....*
(e-g) Text Exercise 6.5 (page 448): *Suppose that the crop researchers in Exercise 6.2 obtained the same value of* $\overline{Y}$....

A4 – 3. Five companies interview two co-op. students. After the interviews, each student selects their preferred employer. Assume that company preference is chosen equiprobably ("at random").

(a) For the case of 5 companies and 2 students, list the 25 possible outcomes (*i.e.*, the 25 points of the sample space for this situation) and then find the probability of each of the following events:
- A – "the first company is never selected";
- B – "the first two companies are never selected";
- C – "no company is selected more than once."

(b) For the case of 5 companies and n students, find the probabilities of events A, B and C if $n \leq 5$.

(c) For the case of k companies and n students, find the probabilities of events A, B and C if $n \leq k$.

A4 – 4. The 13 letters of the word 'ACCOMMODATION' are arranged in an order equiprobably. Find:
(a) the number of *distinguishable* arrangements;
(b) the probability that the letters spell 'ACCOMMODATION';
(c) the probability that the arrangement begins and ends with the *same* letter;
(d) the probability that the As are *not* beside each other.

A4 – 5. A four-digit number is formed by selecting its digits equiprobably *with replacement* from the eight digits 1, 2, ..., 8. Find the probability of each of the following events:
- A – "all the digits have the same *parity* (all odd or all even)";
- C – "none of the digits is a *one*";
- D – "all the digits are *different*";
- E – "all the digits are *equal*";
- F – "all the digits exceed *four*."

A4 – 6. A city block has 22 families living in it, of which 8 are French-speaking. For a sample survey, eight families are selected equiprobably from the block. Find the probability that:
(a) *exactly* two French-speaking families are in the sample;
(b) *at least* two French-speaking families are in the sample.

A4 – 7. In a lottery, each ticket is filled out by the purchaser with six numbers chosen without replacement from 1, 2, ..., 49. The six *winning* numbers are chosen equiprobably without replacement, also from 1, 2, ..., 49; the *order* in which these six numbers are drawn is irrelevant.
(a) To win first prize, a ticket must contain *all* the six numbers drawn; find the probability that a person with one ticket will win first prize.
(b) To win fifth prize, a ticket must contain exactly three of the six numbers drawn; find the probability that a person with one ticket will win fifth prize.
(c) If the lottery is conducted weekly, find the probability that at least one of the six numbers drawn this week was also drawn last week.
(d) If 50 million tickets are sold for a particular draw of the lottery, find the approximate probability that there is at least one winner of first prize.

A4 – 8. Suppose that 75% of all homeowners fertilize (F) their lawns, 60% apply herbicides (H) and 35% apply insecticides (I). In addition, suppose that 20% apply none of these, 30% apply all three, 56% apply herbicides and fertilizer, and 33% apply insecticides and fertilizer. What percentage apply:
(a) herbicides and insecticides;
(b) herbicides and insecticides but *not* fertilizer?

(*continued overleaf*)

A4 – 9. Referring to question **A4 – 5** overleaf on page 0.15, find the probability of each of the following events, which are *combinations* of the five events defined in the earlier Question:

$A \cap D$; $D \cup F$; $A \cup D \cup F$; $(E \cup D) \cap F$; $E \cup (D \cap F)$.

Show the last two of these events on Venn diagrams.

A4 – 10. Two teams play a best-of-seven series. The first two games are to be played on *A*s field, the next three games on *B*s field, and the last two (if needed) on *A*s field. If the probability that *A* wins a game is 0.7 at home and 0.5 away, find the probability that:

(a) *A* wins the series in 4 games; in 5 games;

(b) the series does *not* go to 6 games.

A4 – 11. A fair die is rolled twice. We define the following two events: *F* – "the first roll is a 4"; *T* – "the total is *Y*."

Show that *F* and *T* are probabilistically independent events for $Y = 7$ but *not* for $Y = 8$.

A4 – 12. Electrical transformers are purchased in batches of 20 and a sample of size 5 selected equiprobably is inspected. The batch is accepted if there are no defectives and returned to the supplier if there are two or more. If there is *one* defective in the sample, five more transformers are selected equiprobably from the remainder of the batch and inspected. The batch is then accepted if the second sample contains no defectives; otherwise it is returned. Calculate the probability of acceptance as a function of *d*, the number of defectives in the batch.

A4 – 13. In a university, suppose that 6% of the students are in a program of joint honours statistics and computer science, and that 15% are in honours statistics either by itself or in a joint program. Find the probability that an equiprobably-chosen honours statistics student is also taking honours computer science.

A4 – 14. Suppose that 0.2% of all commercial airline flights should have the takeoff aborted because of equipment malfunction. A warning light in the cockpit indicates the danger 99% of the time when the takeoff should be aborted. However, the light occasionally malfunctions and comes on when there is no danger; suppose that this occurs

(i) 1% (ii) 0.1% (iii) 0.01% of the time.

(a) If the light comes on, find the probability that the takeoff should be aborted in each of the three cases.

(b) What do you conclude from your three probabilities in (a) about the reliability required in a warning system that is widely used to detect an important but rare fault? Explain briefly.

A4 – 15. A researcher wishes to estimate the proportion p of university students who have cheated on an examination. She prepares a box containing 100 cards, 20 of which contain Question *B* and 80 Question *C*.

Question B: Were you born in July or August?

Question C: Have you ever cheated on an examination?

Each student who is interviewed draws a card equiprobably with replacement from the box and answers the question it contains. Since only the student knows which question he or she is answering, confidentiality is assured and so the researcher hopes that the answers will be truthful. It is known that one-sixth of birthdays fall in July or August.

(a) Find an expression for the probability that a student answers 'yes'.

(b) If y of n students answer 'yes', estimate p in terms of y and n.

(c) Find an expression for the proportion of the students who answer 'yes' that are responding to Question *C*.

***A4 – 16.** Three convicts, *A*, *B* and *C*, know that their warder has chosen one of them equiprobably to be shot at dawn. Convict *A* is, naturally enough, curious to know whether he has been chosen. He realizes that the warder would refuse to answer the direct question "*Am I to be shot?*"; however, since at least one of *B* and *C* is to be spared, he believes that the warder would consent to name one of these two who is to be spared. *A* therefore asks the warder, who tells him that *C* is to be spared. Convict *A* goes away very unhappy; before he asked, he had only one chance in three of being shot, but now he knows that *C* is to be spared, his chance of being shot has increased to one in *two*. Explain whether *A*s reasoning is valid.

 *Optional problem for enrichment