University of Waterloo STAT 220 – W. H. Cherry

Assignment 3

A3 – 1. Text Exercise 1.69 (page 85): *Many software packages have "random number generators" that produce .....*
[*We* use the terminology "equiprobable digit generator."]

A3 – 2. (a) If the random variable $Y \sim U(\lambda, \upsilon)$, find the probability that Y lies within 1, within 2 and within 3 s.d.s of its mean.
(b) If the random variable $T \sim Exp(\theta)$, find the probability that T lies within 1, within 2 and within 3 s.d.s of its mean.
(c) Compare and contrast the values you obtain in (a) and (b) with those for the normal distribution in Figure 5.3 of the Course Materials.

A3 – 3. If the random variable $T \sim Exp(\theta)$, predict whether the *median* will be less than, equal to, or greater than the mean; explain your reasoning briefly. Then find the median in terms of θ.

A3 – 4. Suppose that the time from treatment to recurrence of a certain type of cancer can be modelled by an exponential distribution with mean θ. Find the value of θ such that there is a probability of 40% of a recurrence of the cancer within 504 days.

A3 – 5. Suppose that the lifetime, T, of a particular type of light bulb can be modelled by an exponential distribution with mean θ days. The lifetimes (in days) observed for 30 bulbs selected equiprobably ("at random") are shown at the right. The average of these 30 lifetimes, $\bar{t} = 59.6\dot{3}$ days, can be taken as an estimate of the parameter θ.

23	261	87	7	120	14
62	47	225	71	247	21
42	20	5	12	120	11
3	14	71	11	14	11
16	90	1	16	52	95

(a) On the basis of the exponential model, evaluate:
$\Pr(0 \le T \le 40)$; $\Pr(40 < T \le 100)$; $\Pr(100 < T \le 200)$; $\Pr(T > 200)$.
Compare the expected frequencies with the observed frequencies in these four intervals; comment briefly on their agreement.
(b) Find values for a, b, c such that *each* of the intervals $(0, a)$, (a, b), (b, c), (c, ∞) has expected frequency 7.5, and find the corresponding observed frequencies. Does the exponential model fit the data satisfactorily? Explain briefly.

A3 – 6. An investigation of working couples measures the income H of the husband and the income W of the wife in a large number of couples in which both partners are employed. Suppose that you knew the means μ_H and μ_W and the standard deviations σ_H and σ_W of the distributions used to model the two income variables in the population. If it is desired to model couple *total* income in the population, explain whether it is reasonable to take:
(a) the mean of the distribution used to model $H+W$ to be $\mu_H + \mu_W$;
(b) the standard deviation of the distribution used to model $H+W$ to be $\sqrt{\sigma_H^2 + \sigma_W^2}$.

A3 – 7. The weights of the eggs produced by a certain breed of hen can be modelled by a normal distribution with a mean of 65 grams and a standard deviation of 5 grams.
(a) Explain briefly whether a carton containing a dozen eggs can reasonably be considered to be equivalent to a sample of size n = 12 eggs selected equiprobably from the population of all eggs produced by this breed of hen.
(b) Assuming that the assumption of equiprobable selecting (EPS) *is* justified, find the probability that the total weight of the eggs in a carton selected equiprobably is between 750 and 825 grams.
(c) In light of your answer to (a), outline whether your probability in (b) is likely to be clearly below, close to, or clearly above the *true* probability.

A3 – 8. Text Exercise 5.71 (pages 429-430): *A machine fastens plastic screw-on caps on to containers of motor oil. .....*

A3 – 9. Text Exercise 5.38 (pages 411-412): *The design of an electronic circuit calls for a 100-ohm resistor .....*

A3 – 10. Text Exercise 5.30 (page 409): *Judy's doctor is concerned that she may suffer from hypokalemia .....*

A3 – 11. Text Exercise 5.27 (page 409): *A laboratory weighs filters from a coal mine to measure the amount of dust .....*

(*continued overleaf*)

A3 – 12. A sugar distributor filling 2 kg bags of sugar claims that her filling machine delivers 2.02 kg, on average, with a standard deviation of 0.01 kg. A weights-and-measures inspector selects equiprobably four 2-kg bags of the sugar, pools their contents, and finds 7.99 kg. The distributor says: *Oh well, only about one bag in forty will be underweight.* The inspector says: *I understand that, but my observation does not agree with your filling description*. Explain briefly the statistical issues involved in these two statements; your answers should include relevant probability calculations.

A3 – 13. Text Exercise 5.32 (page 410): *A company that owns and services a fleet of cars for its sales force .....*
[but use $\mu = 85{,}000$ km, $\sigma = 7{,}250$ km in (a) and $\overline{y} = 79{,}800$ km in (b)].

A3 – 14. Text Exercise 5.35 (page 411): *The level of nitrogen oxides (NOX) in the exhaust of a particular car model .....*

A3 – 15. The number of traffic accidents per week at a busy intersection varies with average 2.2 and standard deviation 1.4; the data here are *discrete* so that an appropriate probability model would *not* be a normal distribution.

(a) Let the random variable $\overline{Y}$ represent the *average* number of accidents per week at the intersection during a year (52 weeks). What is the approximate distribution of $\overline{Y}$?

(b) Find the approximate probability that $\overline{Y}$ is fewer than 2.

(c) Find the approximate probability that the *total* number of accidents at the intersection in a year is fewer than 100.

A3 – 16. Text Exercise 5.74 (page 430): *In an experiment on learning foreign languages, researchers studied .....*