SEMESTER PROJECTS, PMATH 950, SECTION 6383, FALL 2018

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Abstract. Semester projects for PMath 950(Completely bounded maps), Fall 2018

You should choose one of the topics below, or create your own project and get it approved by me, and write an exposition of your chosen topics. The paper should be in tex and should be at least 5 pages long.

I would like each project to be done by only one student. So once that you are certain of your project, come and claim it. For most of the topics you should plan on meeting with me a couple of times for some guidance and references.

1. Von Neumann's proof and Marshall's theorem

Write an outline of von Neumann's original proof of his famous inequality. Compare and contrast his proof to Marshall's characterization of the extreme points of the unit ball of the disk algebra. Did von Neumann anticipate Marshall's theorem?

2. Wittstock's Matricial sublinear functionals

This project requires the ability to read German. Read Wittstock's paper *Ein operator vertiger Hahn-Banach satz*, give translations of all the key results and definitions into English and present at least one proof.

3. Multivariable von Neumann Inequalities

Give a more current and thorough account of attempts to generalize von Neumann's inequality to several variables than is done in the book. Discuss analogues of these inequalties for domains other than polydisks.

4. Tensor products of Banach spaces(John Dykes)

Give an overview of the theory of tensor products of Banach spaces. Include key results and definitions. Discuss nuclear spaces in this category.

5. Tensor products of operator spaces

Give an overview of the theory of tensor products of operator spaces. Include key results and definitions. Discuss nuclear spaces in this category. Given two C*-algebras, discuss the difference between their maximal C*-tensor product and their maximal operator space tensor product.

6. Tensor products of operator systems(Pawel Sarkowicz)

Give an overview of the theory of tensor products of operator systems. Discuss nuclear spaces in this category. Given two C*-algebras, outline the proof that their maximal C*-tensor product is the same operator system as their maximal operator system tensor product.

7. Choi's theorem and Stinespring's Theorem

Choi's theorem can be viewed as the special case of Stinespring's theorem when the domain of the CP map is a matrix algebra. Derive Choi's theorem as a corollary to Stinespring's theorem, give Choi's simple direct matrix-theoretic proof of his theorem, and also deduce Stinespring's theorem in the case of a matrix algebra as a corollary of Choi's theorem. Include Choi's results characterizing extreme points of the unital completely positive maps.

8. Hilbert C*-modules(Jacob Campbell)

Give a brief introduction to Hilbert C*-modules. Present the various natural operator space structures that can be put on a Hilbert C*-module. Discuss various notions of injectivity and projectivity in the category of Hilbert C*-modules over a fixed C*-algebra and what is known.

9. Injectivity and projectivity in analysis and topology(Benjamin Anderson-Sackaney)

Read the paper of the above title by Hadwin-P. and give a summary of the key results and concepts. Include a proof of Gleason's result on projective covers of topological spaces, that is used in the paper, but not proven there.

10. Automatic complete contractivity(John Swatzky)

Some operator algebras have the property that every contractive homomorphism is automatically completely contractive. The disk algebra is an example that we will cover early on. Another example is the algebra of upper triangular matrices, but it is still unknown if the same is true for the algebra of all upper triangular operators on $\ell^2(\mathbb{N})$. Give a survey of what is known and not known on this topic, include the Foias-Suciu result for $H^\infty(\mathbb{D})$.

11. Schur product maps, graphs and matrix completions

The linear maps given by Schur product against a given matrix are a particularly simple family of maps that have been studied extensively with many connections to combinatorics. Give a survey of some of the results in this area. Include the connections between extensions of these maps, graphs, and partially defined matrices.

12. Duals of operator spaces and the diamond norm(Connor Paul-Paddock)

The dual of an operator space can be endowed with a particular matrix norm structure that makes it into an operator space. Present the basic facts about this *operator space dual*. The dual of M_n is the matrices with the Schatten 1-norm. This space is sometimes denoted S_n. Thus, S_n ha a particular operator space structure. Describe this structure carefully. Prove that the cb-norm of a map $T : S_n \to S_n$, where S_n is endowed with its dual operator space structure, is equal to the $\diamond$-norm introduced in quantum information theory.

13. Min and Max operator space structures(Dan Ursu)

Given a normed space X, in general there are many matrix norms that it can be endowed with to make it into an operator space. The space X, endowed with the smallest and largest such matrix norms is denoted $Min(X)$ and $Max(X)$, respectively. These two operations can be viewed as functors from the category of normed spaces and bounded(respectively, contractive) maps into the category of operator spaces and cb-maps(respectively, contractive maps). Prove some basic facts about these functors and outline what is known in this theory. Include a proof that $Min(X)^* = Max(X^*)$ as operator spaces.

14. Minimal and maximal operator system structures

This project is similar to the last project but for the case of ordered spaces and matrix orders that make them into operator systems.

15. Matrix convex sets(Adam Humeniuk)

Present the definition of matrix convexity and some of the key results. Given a convex set C in a vector space V, show that there is also a minimal and maximal way to make C into a matrix convex set.

16. Complete Pick Kernels

Present the Nevanlinna-Pick theory, its generalizations to the polydiscs and to reproducing kernel HIlbert spaces and the theorem about complete Pick kernels. Include the role of abstract operator algebras. Give some examples from the paper of Mittal-P.

17. The Kadison-Singer Problem(Daniel Peres-Anaya)

This was an open problem for about 50 years and just recently solved. Discuss the problem, and its equivalences, especially to the Feichtinger conjecture, and what we now know to be true.

18. Graphs and Operator systems(Andrej Vukovic)

To every graph on n vertices there is associated an operator subsystem of M_n. Discuss some of the relations between graph parameters like coloring and independence numbers and these operator systems. Mention the work of Weaver and others on Ramsay theory for operator systems that is based on this correspondence.

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