

PMATH 340 Number Theory, Exercises for Chapter 3 (The Group of Units Modulo N)

- 1:** (a) Make a table of powers in \mathbf{Z}_{21} , showing the values of x^k for all $x \in \mathbf{Z}_{21}$ and all $1 \leq k \leq 7$.
(b) Find the order of each element in U_{21} .
(c) Solve $x^{100} = x$ in \mathbf{Z}_{21} .
- 2:** (a) Find 7^{24} , 143^{962} and $1102^{1101} \pmod{1100}$.
(b) Find $4210^{2142} \pmod{6300}$.
- 3:** (a) Find $523^{470^{654}} \pmod{37}$.
(b) Find $60^{59^{58^{57 \dots 1}}} \pmod{19}$
- 4:** (a) Find the largest integer n such that $\psi(n) = 12$.
(b) Find every positive integer n such that $\varphi(n) = 60$.
- 5:** (a) U_{81} is cyclic and is generated by 2, so we have $U_{81} = \{1, 2, 2^2, 2^3, \dots, 2^{53}\}$. Find the number of squares, the number of cubes, and the number of twelfth powers in U_{81} .
(b) U_{128} is generated by -1 and 5 and we have $U_{128} = \{\pm 1, \pm 5, \pm 5^2, \dots, \pm 5^{31}\}$. Find the number of squares, the number of cubes, and the number of twelfth powers in U_{128} .