PMATH 340 Number Theory, Exercises for Chapter 2 (Integers Modulo N)

1: (a) Find the inverse of 178 in $\mathbf{Z}_{365}$.

(b) Solve the linear congruence $356\,x \equiv 28 \pmod{730}$.

2: Solve the following system of linear equations in $\mathbf{Z}_{20}$.

$$\begin{aligned} x - 2y + 3z &= 1 \\ 2x + \ y + 4z &= -2 \\ x + 3y + 7z &= 5 \end{aligned}$$

3: Solve the following system of congruences.

$$\begin{aligned} x^2 &\equiv x + 6 \pmod{10} \\ 2x^3 &\equiv 7 \pmod{9} \\ x &\equiv 11 \pmod{24} \end{aligned}$$

4: Solve $x^3 + 6x \equiv 43 \pmod{792}$.

5: Let $n = p^k$ where p is prime and $k \geq 1$. Let $f(x) = x^3 + 2x^2 - x - 2 = (x-1)(x+1)(x+2)$. Determine the number of solutions in $\mathbf{Z}_n$ to the equation $f(x) = 0$. Express your answer in terms of p and k.