PMATH 340 Number Theory, Exercises for Chapter 1 (The Euclidean Algorithm)

1: Let $a = 17537$, $b = 5434$ and $c = 1482$.

    (a) Find $\gcd(a, b)$ and $\operatorname{lcm}(a, b)$.

    (b) Solve the linear Diophantine equation $ax + by = c$.

2: (a) Find $\sigma(10!)$.

    (b) List all of the positive integers n such that $\sigma(n) = 42$.

    (c) Find the smallest positive integer n such that $\tau(n) = 42$.

    (d) For which positive integers n is $\tau(n)$ odd?

    (e) For which positive integers n is $\sigma(n)$ odd?

3: Let $a = (25)!$ and $b = (5500)^3(1001)^2$.

    (a) Find the prime factorization of a and of b.

    (b) Find the prime factorization of $\gcd(a, b)$ and of $\operatorname{lcm}(a, b)$.

    (c) Find the number of positive factors of b which are not factors of a.

    (d) Find the number of factors (positive or negative) of b which are either perfect squares or perfect cubes (or both).

4: Solve the linear Diophantine equation $8x + 18y + 45z + 30w = 4$.

5: Consider the following system of linear Diophantine equations.

$$5x + y + 4z + w = a$$
$$4y + 6z + 9w = 2$$

    (a) Find all integers a such that the system has a solution.

    (b) Solve the system when $a = 3$.