PMATH 340 Number Theory, Assignment 2 Due: Mon Feb 26, 11:00 pm

1: (a) Find all possible pairs of decimal digits (a, b) such that $99 \big| 38a91b$.

(b) Let $n = a_0 + a_1 \cdot 1000 + a_2 \cdot 1000^2 + \cdots + a_\ell \cdot 1000^\ell$ where $a_\ell \neq 0$ and for each i we have $a_i \in \{0, 1, \cdots, 999\}$. Show that for $d = 7$, 11 and 13 we have

$$d \big| n \iff d \big| \left(a_0 - a_1 + a_2 - a_3 + \cdots + (-1)^\ell a_\ell\right).$$

(c) Show that it is not possible to rearrange the digits of the number 51328167 to form a perfect square or a perfect cube or any higher perfect power.

2: (a) Find 12^{-1} in $\mathbb{Z}_{29}$.

(b) Solve $34x = 18$ in $\mathbb{Z}_{46}$.

(c) In $\mathbb{Z}_{20}$, solve the pair of linear equations

$$\begin{aligned} 7x + 12y &= 6 \\ 6x + 11y &= 13 \end{aligned}$$

3: (a) Solve the pair of congruences $5x = 9 \bmod 14$ and $17x = 3 \bmod 30$.

(b) Solve the congruence $x^2 + x = 38 \bmod 72$.

4: Chinese generals used to count their troops by telling them to form groups of some size n, and then counting the number of troops left over. Suppose there were 5000 troops before a battle, and after the battle it was found that when the troops formed groups of 5 there was 1 left over, when they formed groups of 7 there were none left over, when they formed groups of 11 there were 6 left over, and when they formed groups of 12 there were 5 left over. How many troops survived the battle?