PMATH 340 Number Theory, Assignment 2.5 Not to be handed in

1: (a) Find $10^{50} \bmod 91$.

(b) Find $28^{27^{26}} \bmod 25$.

(c) Find a positive integer k such that the number 3^k ends with the digits 0001.

(d) With the help of the following table of powers of 5 mod 64, solve $11\,x^5 = 17 \bmod 64$.

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5^k	1	5	25	61	49	53	9	45	33	37	57	29	17	21	41	13	1
-5^k	63	59	39	3	15	11	55	19	31	27	7	35	47	43	23	51	63

2: (a) Find a positive integer ℓ and find primes $p_1, p_2, \cdots, p_\ell$ and positive integers $k_1, k_2, \cdots, k_\ell$ such that $U_{675} \cong \mathbb{Z}_{{p_1}^{k_1}} \times \mathbb{Z}_{{p_2}^{k_2}} \times \cdots \times \mathbb{Z}_{{p_\ell}^{k_\ell}}$.

(b) Find the number squares, the number of cubes, and the number of fourth powers in U_{125}.

(c) For $n = 18900$, find the universal exponent $\lambda(n)$ and find the number of elements in U_n of order $\lambda(n)$.