MATH 247 Calculus 3, Assignment 5 Due: Fri Apr 5, 11:00 pm.

1: (a) Find $\displaystyle\iint_D y\,e^x\,dA$ where D is the region in $\mathbb{R}^2$ bounded by $y=0$, $y=x$ and $x+y=2$.

(b) Find $\displaystyle\iint_D \frac{x}{\sqrt{1+x^2+y^2}}\,dA$ where $D=\left\{(x,y)\,\middle|\,0\le x\le 2\,,\ 0\le y\le \tfrac{1}{2}x^2\right\}$.

(c) Find $\displaystyle\iiint_D z\ dV$ where $D=\left\{(x,y,z)\,\middle|\,0\le x\,,\ 0\le y\le \sqrt{x^2+z^2}\,,\ 0\le z\le \sqrt{1-x^2}\right\}$.

2: (a) Find $\displaystyle\iint_D \cos(3x^2+y^2)\,dA$ where $D=\left\{(x,y)\,\middle|\,x^2+\tfrac{1}{3}\,y^2\le 1\right\}$.

(b) Find $\displaystyle\iint_D e^{(y-x)/(y+x)}\,dA$ where D is the quadrilateral with vertices at $(1,1)$, $(2,0)$, $(4,0)$, $(2,2)$.

(c) Find $\displaystyle\iiint_D (x-y)z\,dV$ where $D=\left\{(x,y,z)\,\middle|\,x^2+y^2+z^2\le 4\,,\ z\ge\sqrt{x^2+y^2}\,,\ x\ge 0\right\}$.

3: (a) Find the total charge in the region $D=\left\{(x,y,z)\,\middle|\,\sqrt{\tfrac{1}{3}(x^2+y^2)}\le z\le\sqrt{4-x^2-y^2}\right\}$ where the charge density (charge per unit volume) is given by $f(x,y,z)=x^2$.

(b) Read Definition 7.13 and Note 7.14. Find the mass of the sphere $x^2+y^2+z^2=1$ when the density (mass per unit area) is given by $f(x,y,z)=3-z$ (this is Exercise 7.17).

(c) Find the mass of the curve of intersection of the parabolic sheet $z=x^2$ with the paraboloid $z=2-x^2-2y^2$ when the density (mass per unit length) is given by $f(x,y,z)=|xy|$ (this is Exercise 7.18).

4: Let $f:[a,b]\to[c,d]$ be bijective and decreasing with $f(a)=d$ and $f(b)=c$, and let $g=f^{-1}:[c,d]\to[a,b]$.

(a) Suppose f and g are differentiable and consider the volume of the solid obtained by revolving the region $a\le x\le b$, $c\le y\le f(x)$ about the x-axis. Prove (using theorems from Calculus 2) that when we calculate the volume using polar coordinates for y and z, Fubini's Theorem holds so that

$$\int_{x=a}^{b}\int_{r=c}^{f(x)}\int_{\theta=0}^{2\pi} r\ d\theta\,dr\,dx=\int_{\rho=c}^{d}\int_{\varphi=0}^{2\pi}\int_{x=a}^{g(\rho)}\rho\ dx\,d\varphi\,d\rho.$$

In other words, prove that we obtain the same value using the "discs method" or using the "shells method".

(b) Suppose f and g are continuous and consider the area of the region $a\le x\le b$, $c\le y\le f(x)$. Prove (using theorems from Calculus 2) that Fubini's Theorem holds, that is

$$\int_{x=a}^{b}\int_{y=c}^{f(x)} 1\ dy\,dx=\int_{y=c}^{d}\int_{x=a}^{g(y)} 1\ dx\,dy$$

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