MATH 247 Calculus 3, Assignment 3 Due: Fri Mar 8, 11:00 pm.

1: Find a parametric equation for the tangent line at $(\sqrt{3}, 2, -1)$ to the curve of intersection of the two paraboloids $z = 6 - x^2 - y^2$ and $z = x^2 + y^2 - 4y$ using the following two methods:

(a) Find a parametric equation for the curve of intersection of the two paraboloids.

(b) Find the tangent plane at $(\sqrt{3}, 2, -1)$ to each of the two paraboloids, then find the line of intersection of these two planes.

2: Find an implicit equation (of the form $ax + by + cz = d$) for the tangent plane to the parametric surface $(x, y, z) = f(\theta, \phi) = \big(\sin 2\theta \cos\phi, \sin 2\theta \sin\phi, \cos\theta\big)$ at the point $f\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ using the following two methods:

(a) Find a parametric equation for the tangent plane, then convert it to an implicit equation.

(b) Find an implicit equation for the surface and use it to obtain an equation for the tangent plane.

3: Read Theorem 4.20 (the Chain Rule) and Definition 4.24 (the directional derivative), then solve the following.

(a) Let $(u, v) = g(z) = \big(\sqrt{z - 1}, 5 \ln z\big)$, where $z = f(x, y) = 4x^2 - 8xy + 5y^2$. Use the Chain Rule to find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ at the point $(2, 1)$ (this is Exercise 4.21 in the Lecture Notes).

(b) Let $(x, y) = f(r, \theta) = \big(r\cos\theta\, ,\, r\sin\theta\big)$, let $z = g(x, y)$ and let $z = h(r, \theta) = g\big(f(r, \theta)\big)$. Suppose that $h(r, \theta) = r^2 e^{\sqrt{3}(\theta - \frac{\pi}{6})}$. Find $Dg(\sqrt{3}, 1)$ (this is Exercise 4.22).

(c) Let $f(x, y, z) = x\,\sin(y^2 - 2xz)$ and let $\alpha(t) = \big(\sqrt{t}\, ,\, \frac{1}{2}t\, ,\, e^{(t-4)/4}\big)$. Find the rate of change of f as we move along the curve $\alpha(t)$ when $t = 4$ (this is Exercise 4.25).

4: Read Theorem 4.26 then solve the following.

(a) Consider the surface $z = f(x, y)$ where $f(x, y) = \dfrac{4}{2 + x^4 + x^2 + y^2}$. An ant walks counterclockwise around the curve of intersection of the above surface $z = f(x, y)$ with the cylinder $(x - 2)^2 + y^2 = 5$. Find the value of $\tan\theta$, where θ is the angle (from the horizontal) at which the ant is ascending when it is at the point $\big(1, 2, \frac{1}{2}\big)$.

(b) Consider the surface $z = f(x, y)$ where $f(x, y) = \dfrac{6x}{1 + x^2 + y^2}$. Show that any circle which passes through the points $(1, 0)$ and $(-1, 0)$ is a curve of steepest descent, that is for any point (x, y) on any circle C through the two points $(-1, 0)$ and $(1, 0)$, the slope of C at (x, y) is equal to the slope of the gradient vector $\nabla f(x, y)$.