MATH 247 Calculus 3, Assignment 1 Due Fri Feb 2, 11:00 pm

1: (a) Define $f : \mathbf{R}^2 \to \mathbf{R}$ by $z = f(x,y) = \dfrac{6x}{1+x^2+y^2}$. Sketch the level sets $f(x,y) = c$ for $c = 0, \pm1, \pm2, \pm3$, and the level set $z = f(x,0)$, then sketch the graph of f, that is sketch the surface $z = f(x,y)$.

(b) Define $f : \mathbf{R}^2 \to \mathbf{R}^3$ by $f(r,\theta) = \big(r\cos\theta, r\sin\theta, e^r\big)$. Sketch the range of f, that is sketch the parametric surface $(x,y,z) = f(r,\theta)$.

2: (a) Define $f : \mathbf{R} \to \mathbf{R}^2$ by $f(t) = \big(r(t)\cos t, r(t)\sin t\big)$ where $r(t) = \sin 2t$. Find (with proof) a function $g : \mathbf{R}^2 \to \mathbf{R}$ such that $\text{Range}(f) = \text{Null}(g)$.

(b) Define $g : \mathbf{R}^3 \to \mathbf{R}^2$ by $g(x,y,z) = \big(x^2+y^2-z\,,\ x^2-2x+y^2\big)$, Find (with proof) a function $f : \mathbf{R} \to \mathbf{R}^3$ such that $\text{Range}(f) = \text{Null}(g)$.

3: (a) Let $A = \big\{(x,y) \in \mathbf{R}^2 \big| y > x^2\big\}$. Prove, from the definition of an open set, that A is open in $\mathbf{R}^2$.

(b) Define $f : \mathbf{R} \to \mathbf{R}^2$ by $f(t) = \big(\sin t\,,\ t\,e^t\big)$. Prove that the range of f is not closed in $\mathbf{R}^2$.

(c) Let A be the set of real numbers $x \in [0,1)$ which can be written in base 3 without using the digit 2, or in other words, let A be the set of real numbers of the form $x = \sum_{k=1}^{\infty} \frac{a_k}{3^k}$ with each $a_k \in \{0,1\}$. Determine whether A is open or closed (or neither) in $\mathbf{R}$.

4: (a) Let $A, B \subseteq \mathbf{R}^n$. Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(b) Let $A \subseteq \mathbf{R}^n$. Show that $A' = \overline{A}'$ or, in other words, show that A and $\overline{A}$ have the same limit points.

(c) Let $A, B \subseteq \mathbf{R}^n$ be disjoint closed sets. Show that there exist disjoint open sets $U, V \subseteq \mathbf{R}^n$ with $A \subseteq U$ and $B \subseteq V$.