1: Determine the number of ways the digits $1, 2, 3, \cdots, 8$ can be arranged to form an 8-digit number which is divisible by 11.

2: Find the largest integer $n$ such that $x^8 - x^2$ is a multiple of $n$ for every integer $x$.

3: Let $a_1 = 1$ and for $n \geq 2$ let $a_n = 2a_{n-1} + n$. Find $\lim_{n \to \infty} \frac{a_n}{2^n}$.

4: Let $f$ and $g$ be real-valued functions defined on $[0, 1]$. Suppose that $f(0) > 0$, $f(1) < 0$, $f + g$ is increasing, and $g$ is continuous on $[0, 1]$. Show that $f(x) = 0$ for some $x \in [0, 1]$.

5: Coins are placed on some of the 100 squares in a $10 \times 10$ grid. Every square is next to another square with a coin. Find the minimum possible number of coins. (We say that two squares are next to each other when they share a common edge but are not equal).

6: A set $S$ of positive integers contains exactly 20 multiples of 2, exactly 20 multiples of 3, and exactly 20 multiples of 5. Show that there is a subset of $S$ which contains exactly 10 multiples of 2, exactly 10 multiples of 3, and exactly 10 multiples of 5.
1: Find the largest integer \( n \) such that \( x^8 - x^2 \) is a multiple of \( n \) for every integer \( x \).

2: Show that for all real numbers \( r \) and \( s \), we have \( r + s = 10 \) if and only if there exist \( 2 \times 2 \) matrices \( A \) and \( B \), with real entries, such that \( A \) has eigenvalues 1 and 3, \( B \) has eigenvalues 2 and 4, and \( A + B \) has eigenvalues \( r \) and \( s \).

3: A circle, on the surface of a sphere of surface area 1, divides the sphere into two parts. The smaller of these parts is removed and replaced by a hemisphere. The area of the resulting surface is \( \frac{9}{8} \). Find the surface area of the hemisphere.

4: Coins are placed on some of the 100 squares in a \( 10 \times 10 \) grid. Every square is next to another square with a coin. Find the minimum possible number of coins. (We say that two squares are next to each other when they share a common edge but are not equal).

5: Let \( n \) be a positive integer and let \( p_1, p_2, \ldots, p_n \) be non-constant polynomials with integer coefficients. Show that there exists a positive integer \( k \) such that each \( p_i(k) \) is composite.

6: Let \( a_0 = 1 \) and for \( n \geq 0 \) let \( a_{n+1} = a_n - \frac{1}{2} a_n^2 \). Find \( \lim_{n \to \infty} n a_n \), if it exists.