1: Given two circles $C_1$ and $C_2$ in the plane, find the locus of all points $P$ for which the tangents from $P$ to each of $C_1$ and $C_2$ have equal lengths.

2: How many sets of four distinct points forming the vertices of a trapezoid are there if the points are chosen from the vertices of a regular $n$-gon, where $n$ is an integer $\geq 4$?

3: Let $a_i$ and $c_i$ be positive numbers for $i = 1, 2, \cdots, n$. Prove that

$$\sqrt{(a_1 + c_1)(a_2 + c_2)\cdots(a_n + c_n)} \geq \sqrt[n]{a_1 a_2 \cdots a_n} + \sqrt[n]{c_1 c_2 \cdots c_n}.$$ 

State when the equality is obtained.

4: Find all the integers that can be written in the form

$$\frac{1}{n_1} + \frac{2}{n_2} + \frac{3}{n_3} + \cdots + \frac{1999}{n_{1999}}$$

where $n_1, n_2, \cdots, n_{1999}$ are positive integers.

5: Show that for all positive integers $n$ there exists a positive integer $d$ such that

$$d, 2d, 3d, \cdots, nd$$

are all perfect powers. (A positive integer $m$ is a perfect power if it can be written in the form $j^i$ where $j$ and $i$ are positive integers with $i \geq 2$.)
1: How many sets of four distinct points forming the vertices of a trapezoid are there if the points are chosen from the vertices of a regular \( n \)-gon, where \( n \) is an integer \( \geq 4 \)?

2: Prove or disprove: Suppose \( P(x) \) and \( Q(x) \) are two polynomials in a real variable \( x \) with \( |P(x)|^2 - |Q(x)|^3 = 1 \). Then \( P \) and \( Q \) must be constant polynomials (i.e. of degree zero).

3: Prove or disprove: It is possible to write every real-valued function \( f(x,y,z) \) of three real variables as

\[
f(x,y,z) = \psi(\phi(x,y),z)
\]

where \( \psi \) and \( \phi \) are appropriately chosen real-valued functions of two real variables.

4: A function \( f : \mathbb{R} \to \mathbb{R} \) is said to be convergence preserving (CP) if for every convergent series \( \sum a_n \), the series \( \sum f(a_n) \) also converges.

Prove or disprove: If \( f \) is CP, then there exists a real number \( M \) and an \( \epsilon > 0 \) such that

\[
\frac{f(x)}{x} < M
\]

for all \( 0 < x < \epsilon \).

5: Show that for all positive integers \( n \) there exists a positive integer \( d \) such that

\[d, 2d, 3d, \ldots, nd\]

are all perfect powers. (A positive integer \( m \) is a perfect power if it can be written in the form \( j^i \) where \( j \) and \( i \) are positive integers with \( i \geq 2 \).)