1: Evaluate
\[ \int_{0}^{\pi/2} \frac{1}{1 + \sqrt{\tan x}} \, dx. \]

2: Find all solutions in nonnegative integers \( x \) and \( y \) to the equation
\[ x(x + 1)(x + 2)(x + 3) = \sum_{i=0}^{y} (2i + 1). \]

3: Consider a set of \( n \) points placed arbitrarily in \( d \)-dimensional Euclidean space \( \mathbb{R}^d \). From each point, we draw an arrow to its nearest neighbour among the \( n - 1 \) other points (using the usual Euclidean measure of distance). We assume that each point has a unique nearest neighbour. Two of the \( n \) points are said to be connected if there is an arrow from one to the other. Two points are said to lie in the same cluster if there is a path of connected points from one to the other. Show that each cluster has exactly one reflexive pair of points, i.e. a pair of points joined by arrows in both directions.

4: On a blackboard are written the numbers
\[ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{100}. \]
At each step, two numbers \( a \) and \( b \) are selected arbitrarily from the list, deleted, and replaced by the single number \( a + b + ab \). After 99 steps, one number is left. What are the possible values of this number?

5: Two points, \( A \) and \( B \), are placed in the interior of a circle. The point \( C \) is chosen to lie on the boundary of the circle such that the angle \( ACB \) is maximized. Find general conditions under which the point \( C \) is unique, and construct a method for finding \( C \) under these conditions.
1: Evaluate
\[ \int_0^{\pi/2} \frac{1}{1 + \sqrt{\tan x}} \, dx. \]

2: Let \( f \) be a continuous function of the closed interval \([0, 1]\) into itself. The function \( f \) is said to be an involution if for every point \( x \in [0, 1] \) we have \( f(f(x)) = x \). The involution \( f(x) = x \)
is said to be trivial. Prove that every non-trivial involution has exactly one fixed point, i.e. one point \( y \) with \( f(y) = y \).

3: Let \( x_1, x_2, x_3, \ldots \) be a sequence of positive real numbers. Define
\[ \bar{x}_n = \frac{x_1 + x_2 + \cdots + x_n}{n}. \]
Show that if \( \sum_{n=1}^{\infty} \frac{1}{x_n} < \infty \) then \( \sum_{n=1}^{\infty} \frac{1}{\bar{x}_n} < \infty \).

4: On a blackboard are written the numbers
\[ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{100}. \]
At each step, two numbers \( a \) and \( b \) are selected arbitrarily from the list, deleted, and replaced by the single number \( a + b + ab \). After 99 steps, one number is left. What are the possible values of this number?

5: A black and white cat walks on the plane which is divided into cells by a square grid. Each cell is either black or white. Initially, the cat is sitting on a cell, call it the origin, heading in one of the four compass directions. It proceeds to walk from cell to cell according to the following rule: it moves one cell over in the direction it is heading, when it lands on a white (black) cell it rotates its heading 90 degrees to the right (left) and paints the cell the opposite colour with a brush attached to its tail. Prove that the cat’s trajectory is unbounded.