Notes prepared by
Stanley Burris
March 13, 2001

Feasible Computations

We want to investigate how many steps a computer can execute in a reasonable amount of time. This will give us a rough, but interesting, upper bound on how much we can expect out of an algorithm.

The basic measure of a computational step for a CPU is derived from the frequency of the crystal which provides the clock for the CPU, e.g., a 100 MHz frequency allows 100 million basic steps per second. A simple CPU instruction can be executed in such a step. Commercial workstations at present operate around 200 MHz, and super computers at less than than 400 MHz.

- So one-billionth of a second, called a *nanosecond*, and abbreviated *ns*, is shorter than the time needed to execute a basic cycle on current computers, and we will use it as a bound to the amount of time in a basic step.
- Computers can have large numbers of CPU's working in parallel – at present one million CPU's seems to be a safe upper bound to what is technically feasible.
- Finally let us say that a project which takes a hundred years to complete is "hopeless".

With these numbers we can give an upper bound for the number of basic steps a computer can carry out on a particular project. (The particular numbers are not critical to our discussion – if you want to be more certain, take the thousandfold of each one.)

Now, from the simple fact that

$$\begin{aligned} 1 \text{ year} &= 60^2 \times 24 \times 365 \text{ seconds} \\ &= 3.1536 \times 10^7 \text{ seconds} \end{aligned}$$

we have the following approximate conversion of years to nanoseconds:

# years	>	#nanoseconds
1		3×10^{16}
10		3×10^{17}
100		3×10^{18} (the hopeless case)
1,000		3×10^{19}

and thus a project requiring

$$10^{25} \text{ basic steps is hopeless}$$

(as $10^{25} > (3 \times 10^{18}) \times 10^{6}$). Actually one could probably claim that even 10^{20} basic steps is hopeless for todays computers. In terms of powers of 2 we have

$$2^{85} \text{ basic steps is hopeless}$$

as $2^{85} > 10^{25}$. In particular this says that trying to compute a truth table with 85 variables is just impossible.

While we are looking at upper bounds, let us take a moment to consider the sizes of the popular bounds used in complexity theory, namely $2^n, 2^{2^n}, 2^{2^{2^n}}$, etc.

doubly exponential:	$2^{2^7} = 2^{128}$	so 2^{2^7} basic steps is hopeless
3-fold exponential:	$2^{2^{2^3}} = 2^{256}$	so $2^{2^{2^3}}$ basic steps is hopeless
4-fold exponential:	$2^{2^{2^{2^2}}} = 2^{65,536}$	so $2^{2^{2^{2^2}}}$ basic steps is hopeless