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Rigid Boolean powers

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Shelah proved that in the variety of Boolean algebras there are rigid algebras of every uncountable cardinality (see [2]). We will prove that certain rigid simple algebras $S$ transfer this result to the variety generated by $S$.

If $A$ is an algebra let $\text{Con}(A)$ denote the lattice of congruences of $A$, and let $\text{Aut}(A)$ be the automorphism group of $A$. For $X$ a Boolean space let $A[X]^*$ be the algebra of continuous functions from $X$ to $A$, giving $A$ the discrete topology (this construction is called a bounded Boolean power). For $f, g \in A[X]^*$ let $\ll f = g \gg = \{x \in X \mid fx = gx\}$. $X^*$ is the Boolean algebra of clopen subsets of $X$.

**LEMMA.** For any algebra $A$ and Boolean space² $X$ the map from $(\text{Aut}(A))[X]^*$ to $\text{Aut}(A[X]^*)$ given by $\alpha \mapsto \bar{\alpha}$ where $(\bar{\alpha}f)(x) = (\alpha x)(fx)$, $f \in A[X]^*$, is a group embedding. If this embedding is surjective then $X^*$ must be rigid or $|A| = 1$.

**Proof.** The first part is straight-forward as the mapping is defined component-wise. For the second part note that if $\mu : X \rightarrow X$ is a homeomorphism then the map from $A[X]^*$ to $A[X]^*$ defined by $f \mapsto f \circ \mu$ is an automorphism of $A[X]^*$. If this automorphism is equal to $\bar{\alpha}$ for some $\alpha \in \text{Aut}(A)[X]^*$ then an easy argument shows $\bar{\alpha}$ is the identity map on the constant functions in $A[X]^*$, hence $\bar{\alpha}$ is the identity map on $A[X]^*$, so $\mu$ is the identity map on $X$ or $|A| = 1$. Thus the embedding $\alpha \mapsto \bar{\alpha}$ is surjective implies $X^*$ is rigid, or $|A| = 1$.

It is trivial to show that if $X^*$ is rigid then the map in the above lemma need not be surjective (let $A$ be the group $\mathbb{Z}_2$ and let $X^*$ be an infinite rigid Boolean algebra). In the following we give sufficient conditions on $A$ which ensure that the mapping is surjective, provided $X^*$ is rigid.

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² This lemma actually holds for an arbitrary topological space $X$, where $A[X]^*$ is, as before, the algebra of continuous functions.

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REFERENCES


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