PMath 330 §01 **(1:30 p.m.)** MIDTERM Friday, October 29, 1999

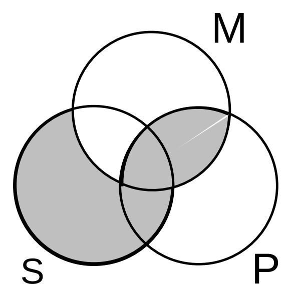
There are 4 pages, with 6 problems.

Problem 1

12 marks

Give the **mood** and **figure** of the syllogism to the right, and fill in the Venn diagram [use more than one if needed] to determine if the syllogism is indeed valid. State whether the syllogism is valid under **Modern Standards** as well as under **Aristotelian Standards**.

All S is M
No P is M
―――――――
Some S is not P.



Mood: EAO **Figure:** 2

Valid (Modern): NO

Valid (Aristotelian): YES

Problem 2

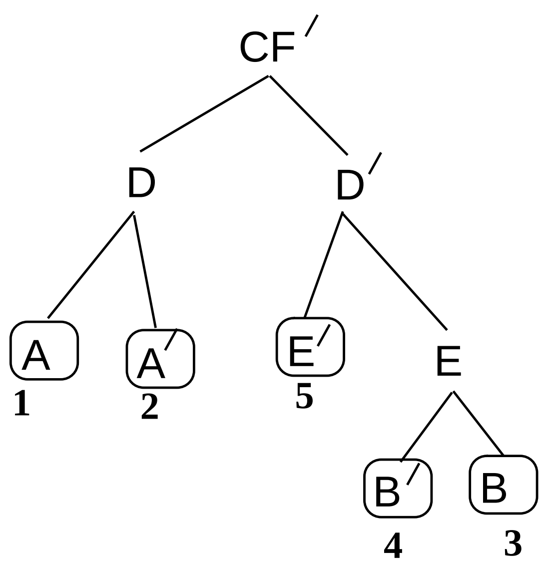
13 marks

Show that the following argument is valid by filling in the Lewis Carroll tree on the right, including appropriate numbers for the boxes.

1. $ACD = 0$
2. $A'DF' = 0$
3. $BCE = 0$
4. $B'D'F' = 0$
5. $D'E'F' = 0$

―――――――

$CF' = 0$



Problem 3

10 marks

Given the following five formulas and their combined truth table

$F_1 : \neg R \vee Q$
$F_2 : R \vee ((P \to R) \leftrightarrow Q)$
$F_3 : \neg Q \to Q$
$F_4 : (Q \leftrightarrow P) \to (Q \wedge (P \leftrightarrow Q))$
$F_5 : \neg R \leftrightarrow Q$

	P	Q	R	F_1	F_2	F_3	F_4	F_5	$F_2 \to F_5$	
1.	1	1	1	1	1	1	1	0	0	
2.	1	1	0	1	0	1	1	1	1	
3.	1	0	1	0	1	0	1	1	1	
4.	1	0	0	1	1	0	1	0	0	
5.	0	1	1	1	1	1	1	0	0	
6.	0	1	0	1	1	1	1	1	1	
7.	0	0	1	0	1	0	0	1	1	
8.	0	0	0	1	0	0	0	0	1	

(a) What is the Conjunctive Normal Form of $F_2 \to F_5$?

$$(\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee \neg R)$$

(b) Is the set $\{\neg F_1, F_2, \neg F_3\}$ satisfiable? (Why?)

YES (Row 3, or Row 7)

(c) Is the argument $F_1, \neg F_3 \therefore F_4$ valid? (Why?)

NO (In Row 8 the premisses are true, the conclusion false.)

Problem 4

12 marks

State the **pigeonhole principle** $\mathbb{P}_2$ (this means there are two pigeonholes) and express it as a collection of clauses. (This was the last topic covered in the lectures on clauses.)

$\mathbb{P}_2$ asserts that if we put three objects into two slots then in some slot there must be two objects; or equivalently, it is not possible to put three objects into two slots with no two objects in the same slot.

The clauses for $\mathbb{P}_2$ are:

$\{P_{i1}, P_{i2}\}$ for $1 \leq i \leq 3$
$\{\neg P_{ik}, \neg P_{jk}\}$ for $1 \leq i < j \leq 3,\ 1 \leq k \leq 2.$

Or, in detail,

$$\begin{array}{lll} \{P_{11}, P_{12}\} & \{P_{21}, P_{22}\} & \{P_{31}, P_{32}\} \\ \{\neg P_{11}, \neg P_{21}\} & \{\neg P_{12}, \neg P_{22}\} & \{\neg P_{11}, \neg P_{31}\} \\ \{\neg P_{12}, \neg P_{32}\} & \{\neg P_{21}, \neg P_{31}\} & \{\neg P_{22}, \neg P_{32}\}. \end{array}$$

Problem 5

12 marks

Fill in the reasons to justify Lemma D.0.10.

> The Frege-Łukasiewicz Propositional Logic
>
> **Propositional Variables:** $P, Q, \ldots$
>
> **Connectives:** $\neg\,, \rightarrow$
>
> **Rule of inference:** (modus ponens) $\dfrac{\mathsf{F},\ \mathsf{F} \rightarrow \mathsf{G}}{\mathsf{G}}$
>
> **Axiom schemata:**
>
> **A1:** $\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$
>
> **A2:** $(\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})) \rightarrow ((\mathsf{F} \rightarrow \mathsf{G}) \rightarrow (\mathsf{F} \rightarrow \mathsf{H}))$
>
> **A3:** $(\neg\,\mathsf{F} \rightarrow \neg\,\mathsf{G}) \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$

Lemma A If F is an axiom then $\vdash \mathsf{F}$.

Lemma B If $\mathsf{F} \in \mathcal{S}$ then $\mathcal{S} \vdash \mathsf{F}$.

Lemma D.0.5 $\vdash \mathsf{F} \rightarrow \mathsf{F}$.

Lemma D.0.6 If $\mathcal{S} \vdash \mathsf{F}$ and $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$, then $\mathcal{S} \vdash \mathsf{G}$.

Lemma D.0.7 If $\mathcal{S} \vdash \mathsf{F}$ and $\mathcal{S} \subseteq \mathcal{S}'$, then $\mathcal{S}' \vdash \mathsf{F}$.

Lemma D.0.8 $\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{G}$ iff $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$.

Lemma D.0.9 If $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$ and $\mathcal{S} \vdash \mathsf{G} \rightarrow \mathsf{H}$, then $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}$.

Lemma D.0.10 If $\mathcal{S} \vdash \mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})$ and $\mathcal{S} \vdash \mathsf{G}$, then $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}$.

Proof:

1.	$\mathcal{S} \vdash \mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})$	*given*
2.	$\mathcal{S} \vdash \mathsf{G}$	*given*
3.	$\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{G}$	$D.0.7,\ 2$
4.	$\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{G} \rightarrow \mathsf{H}$	$D.0.8,\ 1$
5.	$\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{H}$	$D.0.6,\ 3, 4$
6.	$\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}$.	$D.0.8,\ 5$

Problem 6

16 marks

(a) Without assuming any set of connectives is adequate, show that $\mathcal{C} = \{\to, \neg\}$ is adequate.

[Blackboard Note: Assume the standard connectives are adequate.]

$$0 \sim \neg(P \to P)$$

$$1 \sim P \to P$$

$$P \lor Q \sim \neg P \to Q$$

$$P \land Q \sim \neg(P \to \neg Q)$$

$$P \leftrightarrow Q \sim \neg\Big((P \to Q) \to \neg(Q \to P)\Big)$$

$$\neg P \sim \neg P$$

$$P \to Q \sim P \to Q$$

(b) Use (a) to show that the Schröder connective $\curlywedge$ is adequate.

$$\neg P \sim P \curlywedge P$$

$$P \to Q \sim \Big((P \curlywedge P) \curlywedge Q\Big) \curlywedge \Big((P \curlywedge P) \curlywedge Q\Big)$$