PMath 330 §02 **(10:30 a.m.)**MIDTERM Friday, October 29, 1999

**There are 4 pages, with 6 problems.**

## Problem 1

**12 marks**

Give the **mood** and **figure** of the syllogism to the right, and fill in the Venn diagram [use more than one if needed] to determine if the syllogism is indeed valid. State whether the syllogism is valid under **Modern Standards** as well as under **Aristotelian Standards**.

All M is S
No M is P
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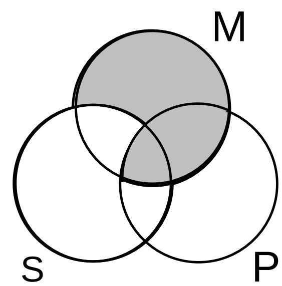
Some S is not P.




**Mood:** EAO **Figure:** 3

**Valid (Modern):** NO

**Valid (Aristotelian):** YES

## Problem 2

**13 marks**

Show that the following argument is valid by filling in the Lewis Carroll tree on the right, including appropriate numbers for the boxes.

1. $CDF = 0$
2. $A'CF' = 0$
3. $BDE = 0$
4. $A'C'E' = 0$
5. $A'B'C' = 0$

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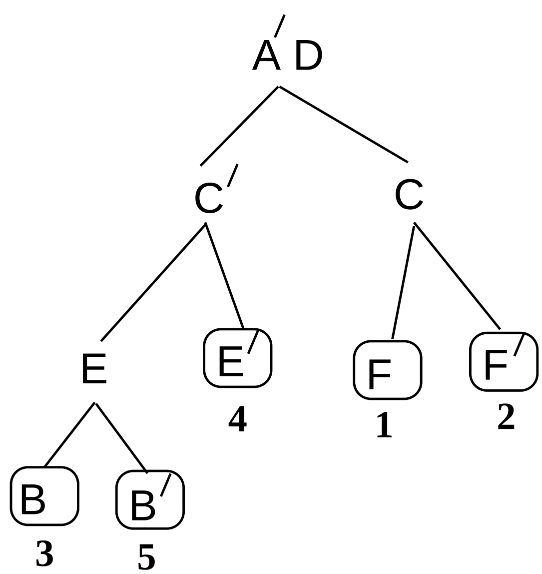
$A'D = 0$



## Problem 3

**10 marks**

Given the following five formulas and their combined truth table

$F_1 : \neg R \vee Q$
$F_2 : R \vee ((P \to R) \leftrightarrow Q)$
$F_3 : \neg Q \to Q$
$F_4 : (Q \leftrightarrow P) \to (Q \wedge (P \leftrightarrow Q))$
$F_5 : \neg R \leftrightarrow Q$

| | $P$ | $Q$ | $R$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_1 \to F_5$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | | |
| 2. | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | | |
| 3. | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | | |
| 4. | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | | |
| 5. | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | | |
| 6. | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | | |
| 7. | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | | |
| 8. | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | | |

**(a)** What is the Conjunctive Normal Form of $F_1 \to F_5$?

$$(\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee R)$$

**(b)** Is the set $\{F_1, \neg F_2, F_3\}$ satisfiable? (Why?)

YES (Row 3)

**(c)** Is the argument $\neg F_1,\ F_3 \therefore F_4$ valid? (Why?)

YES (The premisses are not satisfied in any row.)

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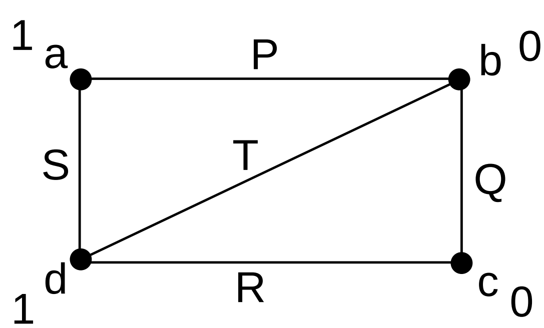
## Problem 4

**14 marks**

Find the graph clauses for the labelled graph on the right, and state (with reasons) whether or not this set of clauses is satisfiable.




| | | | | |
|---|---|---|---|---|
| $Clauses(a)$ : | $\{P, S\}$ | $\{\neg P, \neg S\}$ | | |
| $Clauses(b)$ : | $\{\neg P, Q, T\}$ | $\{P, \neg Q, T\}$ | $\{P, Q, \neg T\}$ | $\{\neg P, \neg Q, \neg T\}$ |
| $Clauses(c)$ : | $\{\neg Q, R\}$ | $\{Q, \neg R\}$ | | |
| $Clauses(d)$ : | $\{R, S, T\}$ | $\{\neg R, \neg S, T\}$ | $\{\neg R, S, \neg T\}$ | $\{R, \neg S, \neg T\}$ |

As the TOTAL CHARGE is 0, this collection of 12 clauses **is satisfiable**.

## Problem 5

**10 marks**

The following gives the first part of the completeness proof for the FŁ proof system. Fill in the reasons to justify Lemma D.0.5.

> The Frege-Łukasiewicz Propositional Logic
>
> **Propositional Variables:** $P, Q, \ldots$
>
> **Connectives:** $\neg, \rightarrow$
>
> **Rule of inference:** (modus ponens) $\dfrac{\mathsf{F},\ \mathsf{F} \rightarrow \mathsf{G}}{\mathsf{G}}$
>
> **Axiom schemata:**
>
> **A1:** $\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$
>
> **A2:** $(\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})) \rightarrow ((\mathsf{F} \rightarrow \mathsf{G}) \rightarrow (\mathsf{F} \rightarrow \mathsf{H}))$
>
> **A3:** $(\neg\mathsf{F} \rightarrow \neg\mathsf{G}) \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$

**Lemma A:** If $\mathsf{F}$ is an axiom then $\vdash \mathsf{F}$.

**Lemma B:** If $\mathsf{F} \in \mathcal{S}$ then $\mathcal{S} \vdash \mathsf{F}$.

**Lemma D.0.5:** $\vdash \mathsf{F} \rightarrow \mathsf{F}$.

**Proof.** The following gives a derivation of $\mathsf{F} \rightarrow \mathsf{F}$:

| | | |
|---|---|---|
| 1. | $\mathsf{F} \rightarrow ((\mathsf{F} \rightarrow \mathsf{F}) \rightarrow \mathsf{F})$ | $A1$ |
| 2. | $(\mathsf{F} \rightarrow ((\mathsf{F} \rightarrow \mathsf{F}) \rightarrow \mathsf{F})) \rightarrow ((\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})) \rightarrow (\mathsf{F} \rightarrow \mathsf{F}))$ | $A2$ |
| 3. | $(\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})) \rightarrow (\mathsf{F} \rightarrow \mathsf{F})$ | $MP\ 1,2$ |
| 4. | $\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})$ | $A1$ |
| 5. | $\mathsf{F} \rightarrow \mathsf{F}$. | $MP\ 3,4$ |

**Problem 6** **16 marks**

**(a)** Without assuming any set of connectives is adequate, show that $\mathcal{C} = \{\wedge,\ \neg\}$ is adequate.

*[Blackboard Note: Assume the standard connectives are adequate.]*

$$
\begin{aligned}
0 &\sim P \wedge \neg P \\
1 &\sim \neg(P \wedge \neg P) \\
P \vee Q &\sim \neg(\neg P \wedge \neg Q) \\
P \to Q &\sim \neg(P \wedge \neg Q) \\
P \leftrightarrow Q &\sim \neg(P \wedge \neg Q) \wedge \neg(\neg P \wedge Q)
\end{aligned}
$$

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$$
\begin{aligned}
\neg P &\sim \neg P \\
P \wedge Q &\sim P \wedge Q
\end{aligned}
$$

**(b)** Use (a) to show that the Sheffer stroke $|$ is adequate.

$$
\begin{aligned}
\neg P &\sim P|P \\
P \wedge Q &\sim (P|Q)|(P|Q)
\end{aligned}
$$