Opp-unification

Suppose our first-order language has a binary relation symbol r and a binary operation symbol g. Show that the two clauses $\{rggxyzguv,\ rguzgugxy\}$ and $\{\neg rgxygyz\}$ are opp-unifiable, and find the most general opp-unifier (σ_1, σ_2). [Change x, y, z to X, Y, Z in the second clause.]

r	g	g	x	y	z	g	u	v								
r	g	u	z	g	u	g	x	y								
r	g	X	Y	g	Y	Z										

$\downarrow \quad u \leftarrow X$

r	g	g	x	y	z	g	X	v								
r	g	X	z	g	X	g	x	y								
r	g	X	Y	g	Y	Z										

$\downarrow \quad X \leftarrow gxy$

r	g	g	x	y	z	g	g	x	y	v						
r	g	g	x	y	z	g	g	x	y	g	x	y				
r	g	g	x	y	Y	g	Y	Z								

$\downarrow \quad Y \leftarrow z$

r	g	g	x	y	z	g	g	x	y	v						
r	g	g	x	y	z	g	g	x	y	g	x	y				
r	g	g	x	y	z	g	z	Z								

$\downarrow \quad z \leftarrow gxy$

r	g	g	x	y	g	x	y	g	g	x	y	v				
r	g	g	x	y	g	x	y	g	g	x	y	g	x	y		
r	g	g	x	y	g	x	y	g	g	x	y	Z				

$\downarrow \quad Z \leftarrow gxy$

r	g	g	x	y	g	x	y	g	g	x	y	v				
r	g	g	x	y	g	x	y	g	g	x	y	g	x	y		
r	g	g	x	y	g	x	y	g	g	x	y	g	x	y		

$\downarrow \quad v \leftarrow gxy$

r	g	g	x	y	g	x	y	g	g	x	y	g	x	y		
r	g	g	x	y	g	x	y	g	g	x	y	g	x	y		
r	g	g	x	y	g	x	y	g	g	x	y	g	x	y		

Thus $\sigma_1 = \begin{pmatrix} x & \leftarrow & x \\ y & \leftarrow & y \\ z & \leftarrow & gxy \\ u & \leftarrow & gxy \\ v & \leftarrow & gxy \end{pmatrix}$ and $\sigma_2 = \begin{pmatrix} x & \leftarrow & gxy \\ y & \leftarrow & gxy \\ z & \leftarrow & gxy \end{pmatrix}$

Resolution Theorem Proving

For f, g unary function symbols and r a binary relation symbol fill in the reasons for the following resolution derivation:

1.	$\{rfgxx\}$	given
2.	$\{\neg rffgg00\}$	given
3.	$\{\neg rxy,\ \neg ryz,\ rxz\}$	given
4.	$\{\neg rxy,\ rfxfy\}$	given
5.	$\{rffgxfx\}$	$1, 4$ *Res.*
6.	$\{\neg rffgg0y,\ \neg ry0\}$	$2, 3$ *Res.*
7.	$\{\neg rfg00\}$	$5, 6$ *Res.*
8.	$\{\ \}$	$1, 7$ *Res.*

The following uses ground clauses from the given clauses of the previous problem. Identify the source of each ground clause (e.g., clause 3) that is an instance of one of the above given clauses, or otherwise justify the resolution step:

1.	$\{rfg00\}$	`Clause 1`
2.	$\{rfgg0g0\}$	*Clause* 1
3.	$\{\neg rffgg00\}$	*Clause* 2
4.	$\{\neg rffgg0fg0,\ \neg rfg00,\ rffgg00\}$	*Clause* 3
5.	$\{\neg rfgg0g0,\ rffgg0fg0\}$	*Clause* 4
6.	$\{\neg rffgg0fg0,\ \neg rfg00\}$	$3, 4$ *Res.*
7.	$\{rffgg0fg0\}$	$2, 5$ *Res.*
8.	$\{\neg rfg00\}$	$6, 7$ *Res.*
9.	$\{\ \}$	$1, 8$ *Res.*

For the language $\{\vee, \wedge, ', 0, 1\}$ of Boolean algebras write down the clauses $\mathrm{AX}_{\equiv}$.

1.	$\{x \equiv x\}$	Reflexive
2.	$\{x \not\equiv y,\ y \equiv x\}$	Symmetric
3.	$\{x \not\equiv y,\ y \not\equiv z,\ x \equiv z\}$	Transitive
4.	$\{x_1 \not\equiv y_1,\ x_2 \not\equiv y_2,\ x_1 \vee x_2 \equiv y_1 \vee y_2\}$	Replacement for $\vee$
5.	$\{x_1 \not\equiv y_1,\ x_2 \not\equiv y_2,\ x_1 \wedge x_2 \equiv y_1 \wedge y_2\}$	Replacement for $\wedge$
6.	$\{x \not\equiv y,\ x' \equiv y'\}$	Replacement for $'$

Convert the following equational argument into a set of clauses such that the argument is valid iff the set of clauses is not satisfiable.

$$x \cdot y \approx u \cdot v \qquad \therefore\ x \cdot y \approx y \cdot x$$

And justify the derivation of the empty clause.

1.	$\{x \equiv x\}$	Reflexive
2.	$\{x \not\equiv y,\ y \equiv x\}$	Symmetric
3.	$\{x \not\equiv y,\ y \not\equiv z,\ x \equiv z\}$	Transitive
4.	$\{x_1 \not\equiv y_1,\ x_2 \not\equiv y_2,\ x_1 \cdot x_2 \equiv y_1 \cdot y_2\}$	Replacement for $\cdot$
5.	$\{x \cdot y \equiv u \cdot v\}$	from the premiss
6.	$\{a \cdot b \not\equiv b \cdot a\}$	from the conclusion
7.	$\{\ \}$	$5, 6$ *Res.*

Expressing facts about N in first-order language

Give formulas or sentences to express the following:

1. The greatest common divisor of x and y is 1

Answer: $\forall u \Big((u|x \ \wedge \ u|y) \ \rightarrow \ u \approx 1 \Big)$

2. x is of the form 17^n

Answer: $\forall u \Big((prime(u) \ \wedge \ u|x) \ \rightarrow \ u \approx 17 \Big)$

3. Every prime of the form $4n + 1$ is a sum of two squares.

Answer: $\forall x \Big(\big(prime(x) \ \wedge \ \exists y \left(x \approx (4 \cdot y) + 1 \right) \big) \ \rightarrow \ \exists u \, \exists v \left(x \approx (u \cdot u) + (v \cdot v) \right) \Big)$

Let A be the sentence $\exists x \forall y \neg (x + y = 0)$.

Let B be the sentence $\forall x \exists y \left[(0 < x) \rightarrow (0 < y) \wedge (y < x) \right]$.

Put a checkmark in each box for which the structure above satisfies the sentence to the left:

	N	**Z**	**Q**	**R**
A	√			
B			√	√
$\neg A \wedge \neg B$		√		
$A \rightarrow B$		√	√	√
$A \leftrightarrow \neg B$	√		√	√