Critical Pairs: Given the pair of term rewrite rules (with disjoint variables)

$$f\underline{gx} \longrightarrow fx \quad \text{and} \quad \underline{ggfu} \longrightarrow ggu$$

find the critical pair that results from unifying the underlined subterms:

Answer: $fgfu, \quad fggu$

Given the pair of term rewrite rules (with disjoint variables)

$$\underline{fgffx} \longrightarrow fx \quad \text{and} \quad gg\underline{fu} \longrightarrow gfu$$

find the critical pair that results from unifying the underlined subterms:

Answer: $ggfx, \quad gfgffx$

Given the pair of term rewrite rules (with disjoint variables)

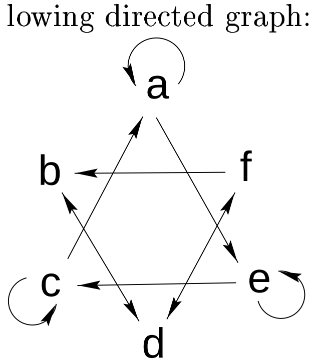
$$x \cdot \underline{(y+z)} \longrightarrow (x \cdot y) + (x \cdot z) \quad \text{and} \quad \underline{(u+v)+w} \longrightarrow u + (v+w)$$

find the critical pair that results from unifying the underlined subterms:

Answer: $(x \cdot (u+v)) + (x \cdot w), \quad x \cdot (u + (v+w))$

Structures

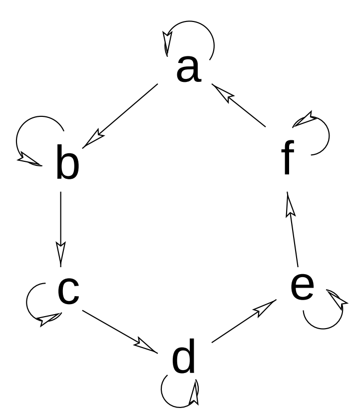
Fill in the table for the following directed graph:



	a	b	c	d	e	f
a	1	0	0	0	1	0
b	0	0	0	1	0	0
c	1	0	1	0	0	0
d	0	1	0	0	0	1
e	0	0	1	0	1	0
f	0	1	0	1	0	0

Draw the directed graph for the following table:

	a	b	c	d	e	f
a	1	1	0	0	0	0
b	0	1	1	0	0	0
c	0	0	1	1	0	0
d	0	0	0	1	1	0
e	0	0	0	0	1	1
f	1	0	0	0	0	1



Herbrand Universe

Suppose that a first-order languge has only a constant symbol 0, a unary function symbol f, and a binary relation symbol r.

Then the third level S_3 of the Herbrand universe is $\{0, f0, ff0, fff0\}$.

Determine the number of **distinct** ground clauses that one can make from the transitivity clause $\{\neg rxy,\ \neg ryz,\ rxz\}$ using S_3.

Answer: 64

Clauses

Let **S** be the structure on $\{0,1,2\}$ with a unary operation defined by $x' = 2x+1 \pmod 3$, and let $a \,\square\, b$ mean that a and b have the same parity. Given the literals

$$L_1 = \neg\,(x' \,\square\, y) \qquad L_2 = x \,\square\, y'$$

determine all pairs (a,b) that satisfy the clause $C = \{L_1, L_2\}$ in **S** by filling in the following:

x	y	x'	y'	$x' \,\square\, y$	$\neg\,(x' \,\square\, y)$	$x \,\square\, y'$	C
0	0	1	1	0	1	0	1
0	1	1	0	1	0	1	1
0	2	1	2	0	1	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	1	0	1
1	2	0	2	1	0	0	0
2	0	2	1	1	0	0	0
2	1	2	0	0	1	1	1
2	2	2	2	1	0	1	1

Does **S** satisfy the clause C? No