Unification

In each of the following you are given a pair of terms that you are to test for being unifiable, and if they are unifiable, give the most general unifier.

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(1) $x + (1 + (x + z))$ and $0 + (u + (v + (u + v)))$

+	x	+	1	+	x	z										
+	0	+	u	+	v	+	u	v								

$\downarrow \quad x \leftarrow 0$

+	0	+	1	+	0	z										
+	0	+	u	+	v	+	u	v								

$\downarrow \quad u \leftarrow 1$

+	0	+	1	+	0	z										
+	0	+	1	+	v	+	1	v								

$\downarrow \quad v \leftarrow 0$

+	0	+	1	+	0	z										
+	0	+	1	+	0	+	1	0								

$\downarrow \quad z \leftarrow 1 + 0$

+	0	+	1	+	0	+	1	0								
+	0	+	1	+	0	+	1	0								

The terms are unifiable, and the most general unifier is $\mu = \begin{pmatrix} x & \leftarrow & 0 \\ u & \leftarrow & 1 \\ v & \leftarrow & 0 \\ z & \leftarrow & 1+0 \end{pmatrix}$

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(2) $x + (y \cdot (z + u))$ and $(u + (u \cdot v)) + ((v + u) \cdot u))$

+	x	·	y	+	z	u										
+	+	u	·	u	v	·	+	v	u	u						

$\downarrow \quad x \leftarrow u + (u \cdot v)$

+	+	u	·	u	v	·	y	+	z	u						
+	+	u	·	u	v	·	+	v	u	u						

$\downarrow \quad y \leftarrow v + u$

+	+	u	·	u	v	·	+	v	u	+	z	u				
+	+	u	·	u	v	·	+	v	u	u						

The terms are not unifiable. (The CSC fails.)

(3) $u \cdot (x + (y \cdot (z + w))$ and $u \cdot ((u + (u \cdot v)) + ((v + u) \cdot u'))$

·	u	+	x	·	y	+	z	w								
·	u	+	+	u	·	u	v	·	+	v	u	′	u			

$\downarrow \quad x \leftarrow u + (u \cdot v)$

·	u	+	+	u	·	u	v	·	y	+	z	w				
·	u	+	+	u	·	u	v	·	+	v	u	′	u			

$\downarrow \quad y \leftarrow v + u$

·	u	+	+	u	·	u	v	·	+	v	u	+	z	w		
·	u	+	+	u	·	u	v	·	+	v	u	′	u			

The terms are not unifiable. (The CSC fails.)

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(4) $(x + (y + z)) + y$ and $u + (z + (v + v))$

+	+	x	+	+	y	z	y									
+	u	+	z	+	v	v										

$\downarrow \quad u \leftarrow x + (y + z)$

+	+	x	+	y	z	y										
+	+	x	+	y	z	+	z	+	v	v						

$\downarrow \quad y \leftarrow z + (v + v)$

+	+	x	+	+	z	+	v	v	z	+	z	+	v	v		
+	+	x	+	+	z	+	v	v	z	+	z	+	v	v		

The terms are unifiable and the most general unifier is $\mu = \begin{pmatrix} u & \leftarrow & x + ((z + (v + v)) + z) \\ y & \leftarrow & z + (v + v) \end{pmatrix}$

For the Normal Form TRS given by

$$\mathcal{R} \approx \{fffx \longrightarrow ffx\}$$

find the normal forms for the following terms:

Term	Normal Form
fx	fx
ffy	ffy
$fffz$	ffz
$ffffu$	ffu
$fffffv$	ffv

For the Normal Form TRS given by

$$\mathcal{R} \approx \{fgx \longrightarrow gfx,\ ffx \longrightarrow fx\}$$

find the normal forms for the following terms:

Term	Normal Form
$gfgx$	$ggfx$
$gfffgy$	$ggfy$
$gfgfz$	$ggfz$
$ffggu$	$ggfu$
$fgfgfv$	$ggfv$

For the Normal Form TRS given by

$$\mathcal{R} \approx \{(x+y)+z \longrightarrow x+(y+z)\}$$

find the normal forms for the following terms:

Term	Normal Form
$(x+x)+x$	$x+(x+x)$
$(x+u)+(y+v)$	$x+(u+(y+v))$
$(x+(u+v))+(v+u)$	$x+(u+(v+(v+u)))$
$((x+w)+(x+u))+y$	$x+(w+(x+(u+y)))$
$(x+y)+((y+z)+(z+w))$	$x+(y+(y+(z+(z+w))))$

Indicate why the TRS

$$\mathcal{R} \approx \{x + y \longrightarrow x \cdot z\}$$

is not terminating for the term $x + y$ by filling in a few steps of

$$x + y \longrightarrow_{\mathcal{R}} x \cdot (x + y) \longrightarrow_{\mathcal{R}} x \cdot (x \cdot (x + y)) \longrightarrow_{\mathcal{R}} x \cdot (x \cdot (x \cdot (x + y)))$$

Indicate why the TRS

$$\mathcal{R} \approx \{x + y \longrightarrow y + x\}$$

is not terminating for the term $x + y$ by filling in a few steps of

$$x + y \longrightarrow_{\mathcal{R}} y + x \longrightarrow_{\mathcal{R}} x + y \longrightarrow_{\mathcal{R}} y + x$$

Indicate why the terminating TRS

$$\mathcal{R} \approx \{fgx \longrightarrow fx,\ gfx \longrightarrow x\}$$

is not a normal form TRS by giving two different terminal forms:

Term	Terminal Form
$fgfx$	ffx
$fgfx$	fx

Indicate why the terminating TRS

$$\mathcal{R} \approx \{x + (y + z) \longrightarrow z + x\}$$

is not a normal form TRS by giving two different terminal forms:

Term	Terminal Form
$x + (u + (v + w))$	$u + x$
$x + (u + (v + w))$	$(v + w) + x$