A Resolution Derivation

Given the collection of 8 clauses

$$a.\ \{P,Q\} \quad b.\ \{P,\neg S\} \quad c.\ \{Q,\neg R\} \quad d.\ \{R,\neg S\}$$
$$e.\ \{\neg P,S\} \quad f.\ \{\neg Q,R\} \quad g.\ \{\neg R,S\} \quad h.\ \{\neg P,\neg Q\}$$

fill in the reasons for the following resolution derivation:

1.	$\{\neg P,S\}$	(e)	8.	$\{\neg R,S\}$	(g)
2.	$\{P,Q\}$	(a)	9.	$\{\neg Q,S\}$	7, 8
3.	$\{Q,S\}$	1, 2	10.	$\{Q,\neg R\}$	(c)
4.	$\{P,\neg S\}$	(b)	11.	$\{R,\neg S\}$	(d)
5.	$\{\neg P,\neg Q\}$	(h)	12.	$\{Q,\neg S\}$	10, 11
6.	$\{\neg Q,\neg S\}$	4, 5	13.	$\{S\}$	3, 9
7.	$\{\neg Q,R\}$	(f)	14.	$\{\neg S\}$	6, 12
			15.	$\{\ \}$	13, 14

Is it possible to find an assignment of truth values for the propositional variables P, Q, R, S that will satisfy the original eight clauses? `NO`

Given the collection $\mathcal{S}$ of 6 clauses

$$1.\ \{P,Q\} \quad 2.\ \{P,\neg S\} \quad 3.\ \{Q,\neg R\}$$
$$4.\ \{R,\neg S\} \quad 5.\ \{\neg P,S\} \quad 6.\ \{\neg Q,R\}$$

fill in the reasons for the following resolution steps:

7.	$\{Q,S\}$	1, 5	14.	$\{\neg P,R\}$	4, 5
8.	$\{P,R\}$	1, 6	15.	$\{Q,R\}$	1, 14
9.	$\{P,\neg P\}$	2, 5	16.	$\{\neg P,Q\}$	3, 14
10.	$\{S,\neg S\}$	2, 5	17.	$\{R,S\}$	5, 8
11.	$\{Q,\neg S\}$	3, 4	18.	$\{Q\}$	7, 11
12.	$\{R,\neg R\}$	3, 6	19.	$\{R\}$	8, 14
13.	$\{Q,\neg Q\}$	3, 6			

Can you obtain any other clauses by resolution? `No`

What does this say about the satisfiability of $\mathcal{S}$? $\mathcal{S}$ `is satisfiable.`

Apply the Davis-Putnam Procedure to the First Problem, showing just the $\mathcal{S}_i'$ and $\mathcal{U}_i$ steps (as done for the resolution on Q below).

Resolution on Q:

$$\mathcal{S}_1': \overset{(1)}{\{P, Q\}}\ \{P, \neg S\}\ \overset{(2)}{\{Q, \neg R\}}\ \{R, \neg S\}\ \{\neg P, S\}\ \overset{(3)}{\{\neg Q, R\}}\ \{\neg R, S\}\ \overset{(4)}{\{\neg P, \neg Q\}}$$

$$\mathcal{U}_1: \overset{(1,3)}{\{P, R\}}\ \overset{(1,4)}{\{P, \neg P\}}\ \overset{(2,3)}{\{R, \neg R\}}\ \overset{(2,4)}{\{\neg P, \neg R\}}$$

Resolution on R:

$$\mathcal{S}_2': \{P, \neg S\}\ \overset{(1)}{\{R, \neg S\}}\ \{\neg P, S\}\ \overset{(2)}{\{\neg R, S\}}\ \overset{(3)}{\{P, R\}}\ \overset{(4)}{\{\neg P, \neg R\}}$$

$$\mathcal{U}_2: \overset{(1,2)}{\{S, \neg S\}}\ \overset{(1,4)}{\{\neg P, \neg S\}}\ \overset{(2,3)}{\{P, S\}}\ \overset{(3,4)}{\{P, \neg P\}}$$

Resolution on P:

$$\mathcal{S}_3': \overset{(1)}{\{P, \neg S\}}\ \overset{(2)}{\{\neg P, S\}}\ \overset{(3)}{\{\neg P, \neg S\}}\ \overset{(4)}{\{P, S\}}$$

$$\mathcal{U}_3: \overset{(1,2)}{\{S, \neg S\}}\ \overset{(1,3)}{\{\neg S\}}\ \overset{(2,4)}{\{S\}}$$

Resolution on S:

$$\mathcal{S}_4': \overset{(1)}{\{S\}}\ \overset{(2)}{\{\neg S\}}$$

$$\mathcal{U}_4: \{\ \}$$

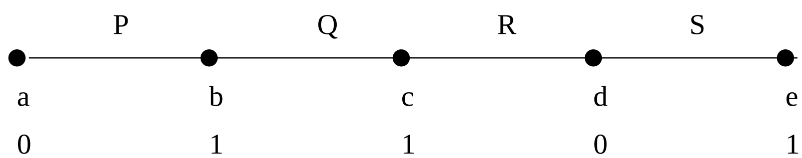
Given the collection of five Horn clauses

$$1.\ \{P, \neg Q\} \quad 2.\ \{\neg P, S\} \quad 3.\ \{\neg R, \neg S\} \quad 4.\ \{Q\} \quad 5.\ \{R\}$$

find all clauses that can be derived using unit resolution:

	Clause	Reason
6.	$\{P\}$	$1, 4$
7.	$\{\neg S\}$	$3, 5$
8.	$\{S\}$	$2, 6$
9.	$\{\neg P\}$	$2, 7$
10.	$\{\neg R\}$	$3, 8$
11.	$\{\neg Q\}$	$1, 9$
12.	$\{\ \}$	$7, 8$ (*or* $6, 9$ *or* $4, 11$ *or* $5, 10$)

Write out the graph clauses associated with the labelled graph:



Clauses(a) $\{\neg P\}$

Clauses(b) $\{P,\ Q\}$ $\{\neg P,\ \neg Q\}$

Clauses(c) $\{Q,\ R\}$ $\{\neg Q,\ \neg R\}$

Clauses(d) $\{\neg R,\ S\}$ $\{R,\ \neg S\}$

Clauses(e) $\{S\}$

Use Tseitin's theorem to determine if this collection of clauses is satisfiable.

ANS: `As the total charge is 1 the collection is not satisfiable.`

Consider the argument:

$$\begin{array}{ll} \mathsf{F}_1: & P \to \neg(Q \to R) \\ \mathsf{F}_2: & (Q \to P) \to (R \to S) \\ \hline \mathsf{F}: & (P \to Q) \land (R \to S) \end{array}$$

Give a conjunctive form for each of the following formulas:

F_1 : $(\neg P \lor Q) \land (\neg P \lor \neg R)$

F_2 : $(Q \lor \neg R \lor S) \land (\neg P \lor \neg R \lor S)$

$\neg \mathsf{F}$: $(P \lor R) \land (P \lor \neg S) \land (\neg Q \lor R) \land (\neg Q \lor \neg S)$

From this derive a set $\mathcal{S}$ of clauses such that $\mathsf{F}_1, \mathsf{F}_2 \therefore \mathsf{F}$ is valid iff $\neg\,\mathsf{Sat}(\mathcal{S})$.

$\mathcal{S}$ has the clauses:

1.	$\{\neg P, Q\}$	2.	$\{\neg P, \neg R\}$
3.	$\{Q, \neg R, S\}$	4.	$\{\neg P, \neg R, S\}$
5.	$\{P, R\}$	6.	$\{P, \neg S\}$
7.	$\{\neg Q, R\}$	8.	$\{\neg Q, \neg S\}$