A Combined Truth Table

1: $\neg Q \wedge P$
2: $Q \wedge (P \wedge R)$
3: $(S \leftrightarrow P) \rightarrow \neg (R \vee S)$
4: $(R \rightarrow Q) \wedge (R \rightarrow P)$
5: $(Q \wedge P) \leftrightarrow (Q \rightarrow \neg S)$
6: $Q \rightarrow \neg (Q \wedge S)$
7: $\neg R \leftrightarrow P$
8: $\neg (P \rightarrow Q)$
9: $R \vee \neg (R \wedge (P \rightarrow R))$
10: $S \leftrightarrow (S \wedge P)$
11: $(R \rightarrow S) \wedge \neg (\neg R \vee S)$
12: $(S \wedge (P \wedge Q)) \wedge R$

	P	Q	R	S	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	0	1	0	1	0	0	0	0	1	1	0	1
2	1	1	1	0	0	1	1	1	1	1	0	0	1	1	0	0
3	1	1	0	1	0	0	0	1	0	0	1	0	1	1	0	0
4	1	1	0	0	0	0	1	1	1	1	1	0	1	1	0	0
5	1	0	1	1	1	0	0	0	0	1	0	1	1	1	0	0
6	1	0	1	0	1	0	1	0	0	1	0	1	1	1	0	0
7	1	0	0	1	1	0	0	1	0	1	1	1	1	1	0	0
8	1	0	0	0	1	0	1	1	0	1	1	1	1	1	0	0
9	0	1	1	1	0	0	1	0	1	0	1	0	1	0	0	0
10	0	1	1	0	0	0	0	0	0	1	1	0	1	1	0	0
11	0	1	0	1	0	0	1	1	1	0	0	0	1	0	0	0
12	0	1	0	0	0	0	1	1	0	1	0	0	1	1	0	0
13	0	0	1	1	0	0	1	0	0	1	1	0	1	0	0	0
14	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0	0
15	0	0	0	1	0	0	1	1	0	1	0	0	1	0	0	0
16	0	0	0	0	0	0	1	1	0	1	0	0	1	1	0	0

- Which of the formulas 1–12 are **truth equivalent**? 1 and 8
- Which of the formulas 1–12 are **tautologies**? 9
- Which of the formulas 1–12 are **contradictions**? 11
- Determine if the following arguments are **valid**. If not, cite the number of a row of the truth table that refutes the argument.

  (a) 3,7,10 $\therefore$ 4 ANS Yes (b) 3,4,5,6,7,9,10 $\therefore$ 8 ANS No (Row 4)

- Determine if the following collections of formulas are **satisfiable**. If so, cite the number of a row of the truth table that satisfies them.

  (a) 1,3,4 ANS Yes (Row 8) (b) 4,5,6,7,8 ANS No

- Find the **disjunctive normal form** of 8 (with respect to the variables P, Q, R, S).
  $(P \wedge \neg Q \wedge R \wedge S) \vee (P \wedge \neg Q \wedge R \wedge \neg S) \vee (P \wedge \neg Q \wedge \neg R \wedge S) \vee (P \wedge \neg Q \wedge \neg R \wedge \neg S)$
- Find the **conjunctive normal form** of 6 (with respect to the variables P, Q, R, S).
  $(\neg P \vee \neg Q \vee \neg R \vee \neg S) \wedge (\neg P \vee \neg Q \vee R \vee \neg S) \wedge (P \vee \neg Q \vee \neg R \vee \neg S) \wedge (P \vee \neg Q \vee R \vee \neg S)$

Translate the following argument (of Lewis Carroll) into propositional formulas. Then by using the premisses, or equivalent propositions, fill in the sequence of implications that shows the conclusion is valid.

1. Animals, that do not kick, are always unexcitable.
2. Donkeys have no horns.
3. A buffalo can always toss one over a gate.
4. No animals that kick are easy to swallow.
5. No hornless animals can toss one over a gate.
6. All animals are excitable, except perhaps buffaloes.

Therefore, donkeys are not easy to swallow.

The universe of discourse is "animals". Use the following:

T: x can always toss one over a gate
B: x is a buffalo
D: x is a donkey
S: x is easy to swallow
E: x is excitable
H: x is horned
K: x kicks

1. $\neg K \to \neg E$
2. $D \to \neg H$
3. $B \to T$
4. $K \to \neg S$
5. $\neg H \to \neg T$
6. $\neg B \to E$

$D \to \neg S$

$$D \to \underline{\neg H} \to \underline{\neg T} \to \underline{\neg B} \to \underline{E} \to \underline{K} \to \neg S.$$

Provide reasons for the following items from Appendix D.

Lemma D.0.12 $\vdash \neg\neg F \to F$.

Proof.

1.		$\vdash$	$\neg\neg F \to (\neg F \to \neg\neg\neg F)$	D.0.11
2.		$\vdash$	$(\neg F \to \neg\neg\neg F) \to (\neg\neg F \to F)$	A3
3.		$\vdash$	$\neg\neg F \to (\neg\neg F \to F)$	1,2 D.0.9
4.	$\neg\neg F$	$\vdash$	$\neg\neg F \to F$	3 D.0.8
5.	$\neg\neg F$	$\vdash$	$\neg\neg F$	Lemma B
6.	$\neg\neg F$	$\vdash$	F	4,5 D.0.6
7.		$\vdash$	$\neg\neg F \to F$.	6 D.0.8

Lemma D.0.14 $\vdash (F \to G) \to (\neg G \to \neg F)$.

Proof.

1.		$\vdash$	$\neg\neg F \to F$	D.0.12
2.	$F \to G$	$\vdash$	$\neg\neg F \to F$	1 D.0.7
3.	$F \to G$	$\vdash$	$F \to G$	Lemma B
4.	$F \to G$	$\vdash$	$\neg\neg F \to G$	2,3 D.0.9
5.		$\vdash$	$G \to \neg\neg G$	D.0.13
6.	$F \to G$	$\vdash$	$G \to \neg\neg G$	5 D.0.7
7.	$F \to G$	$\vdash$	$\neg\neg F \to \neg\neg G$	4,6 D.0.9
8.	$F \to G$	$\vdash$	$(\neg\neg F \to \neg\neg G) \to (\neg G \to \neg F)$	Lemma A (Ax. 3)
9.	$F \to G$	$\vdash$	$\neg G \to \neg F$	7,8 D.0.6
10.		$\vdash$	$(F \to G) \to (\neg G \to \neg F)$.	9 D.0.8