## PMath 330 Assignment $2 \quad$ Solutions

Translate each of the following four syllogisms into equational arguments.
(1) All M is P. $\quad M P^{\prime}=0$
$\frac{\text { All } \mathrm{S} \text { is M. }}{\text { All } \mathrm{S} \text { is } \mathrm{P} .} \frac{S M^{\prime}=0}{S P^{\prime}=0}$

| (2) All M is P. | $M P^{\prime}=0$ |
| :--- | ---: |
| No S is M. | $S M=0$ |
| No S is P. | $S P=0$ |

(3) All P is M .
$P M^{\prime}=0$
$\frac{\text { No } \mathrm{M} \text { is } \mathrm{S} .}{} \begin{array}{r}\text { No } \mathrm{S} \text { is } \mathrm{P} .\end{array} \quad \begin{array}{r}M S=0 \\ =0\end{array}$
(4) No M is P. $\quad M P=0$

$\frac{\text { All S is M. }}{\text { No S is P. }} \quad$| $S M^{\prime}=0$ |
| ---: |
| $S P=0$ |

(a)

(d)

(b)

(c)




Which of the six diagrams above could qualify as Venn diagrams?
Answer:
(b), (f)

The Expansion Theorem
For the formula $F(A, B)=(A B)^{\prime}$ carry out the following computations to calculate the expansion of $F(A, B)$ on $A, B$ :

|  | Full Expression | Value |
| :--- | :--- | :--- |
| $F(1,1)=$ | $(11)^{\prime}$ | $=0$ |
| $F(1,0)=$ | $(10)^{\prime}$ | $=1$ |
| $F(0,1)=$ | $(01)^{\prime}$ | $=1$ |
| $F(0,0)=$ | $(00)^{\prime}$ | $=1$ |

Thus expanding on $A, B$ gives: $F(A, B)=\underline{A B^{\prime} \cup A^{\prime} B \cup A^{\prime} B^{\prime}}$

For the formula $F(A, B, C)=(A B)^{\prime} \cup(B C)$ carry out the following computations to calculate the expansion of $F(A, B, C)$ on $A$ :

|  | Full Expression |  | Simplified |
| :---: | :---: | :---: | :---: |
| $F(1, B, C)=$ | $(1 B)^{\prime} \cup(B C)$ | $=$ | $B^{\prime} \cup C$ |
| $F(0, B, C)=$ | $(0 B)^{\prime} \cup(B C)$ | $=$ | 1 |

Expanding on $A$ gives: $F(A, B, C)=\underline{\left(B^{\prime} \cup C\right) A \cup A^{\prime}}$

For the formula $F(A, B, C)=(A B)^{\prime} \cup(B C)$ carry out the following computations to calculate the expansion of $F(A, B, C)$ on $B, C$ :

|  | Full Expression |  | Simplified |
| :--- | :---: | :--- | :---: |
| $F(A, 1,1)=$ | $(A 1)^{\prime} \cup(11)$ | $=$ | 1 |
| $F(A, 1,0)=$ | $(A 1)^{\prime} \cup(10)$ | $=$ | $A^{\prime}$ |
| $F(A, 0,1)=$ | $(A 0)^{\prime} \cup(01)$ | $=$ | 1 |
| $F(A, 0,0)=$ | $(A 0)^{\prime} \cup(00)$ | $=$ | 1 |

Expanding on $B, C$ gives: $F(A, B, C)=\underline{B C \cup A^{\prime} B C^{\prime} \cup B^{\prime} C \cup B^{\prime} C^{\prime}}$

## Elimination

For the formula $E(A, B, C)=\left(A\left(A^{\prime} \cup B\right)^{\prime} C^{\prime}\right)^{\prime}$ carry out the following computations to eliminate $A$ from the equation $E(A, B, C)=0$ :

|  | Full Expression |  | Simplified |
| :--- | :---: | :---: | :---: |
| $E(1, B, C)=$ | $\left(1\left(1^{\prime} \cup B\right)^{\prime} C^{\prime}\right)^{\prime}$ | $=$ | $B \cup C$ |
| $E(0, B, C)=$ | $\left(0\left(0^{\prime} \cup B\right)^{\prime} C^{\prime}\right)^{\prime}$ | $=$ | 1 |

Eliminating $A$ gives (simplify first!): $\quad B \cup C=0$

## PMath 330 Assignment 2 Solutions

For the formula $E(A, B, C, D)=(A \cup B)(C \cup D)$ carry out the following computations to eliminate $B, C$ from the equation $E(A, B, C, D)=0$ :

| Full Expression |  | Simplified |
| :---: | :---: | :---: | :---: |
| $E(A, 1,1, D)=(A \cup 1)(1 \cup D)$ | $=$ | 1 |
| $E(A, 1,0, D)=(A \cup 1)(0 \cup D)$ | $=$ | $D$ |
| $E(A, 0,1, D)=(A \cup 0)(1 \cup D)$ | $=$ | $A$ |
| $E(A, 0,0, D)=(A \cup 0)(0 \cup D)$ | $=$ | $A D$ |

Eliminating $B, C$ gives (simplify first!): $\quad A D=0$

Fill in the following tree to give a proof of the validity of the argument using the method of Lewis Carroll. Be sure to give the number of the reason for each boxed letter.
$1 \mathrm{ACL}=0$
$2 A^{\prime} I^{\prime} M^{\prime}=0$
$3 \mathrm{GIO}^{\prime}=0$
$4 A^{\prime} C M=0$

| $5 C^{\prime} I^{\prime} O^{\prime}=0$ |
| :---: |
| $L O^{\prime} G=0$ |



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